

**Benson, David J.; Smith, Stephen D.**

**Classifying spaces of sporadic groups.** (English) Zbl 1135.20001

*Mathematical Surveys and Monographs* 147. Providence, RI: American Mathematical Society (AMS) (ISBN 978-0-8218-4474-8/hbk). xvi, 289 p. (2008).

Determining the cohomology of sporadic groups represents an area of considerable current interest, and such study combines methods of algebraic topology with techniques from finite group theory. This book provides each sporadic group with a description of a 2-completed classifying space in terms of classifying spaces of suitable 2-local subgroups, which leads to an additive decomposition of the mod 2 group cohomology in particular.

The book consists of two parts together with Chapter 1 devoted to a brief summary of the main results. Chapter 2 recalls some basics of the group cohomology of a finite group  $G$ , including aspects of the topological approach via the classifying space  $BG$ , and of approximating  $BG$  via the Borel construction on a suitable  $G$ -space. In Chapter 3 these topological spaces are viewed as simplicial sets, which provides a general context for Chapter 4 devoted to Bousfield-Kan completions and homotopy colimits. Chapter 5 discusses some aspects of the modern literature on homotopy and homology decompositions for  $BG$ , which leads up to Dwyer's notion of homology decomposition [as given in *W. G. Dwyer, H.-W. Henn, Homotopy theoretic methods in group cohomology. Advanced Courses in Mathematics – CRM Barcelona. Basel: Birkhäuser (2001; Zbl 1047.55001)*]. Part 1 concludes with Chapter 6, which reviews some concepts from the group-theoretic literature on geometries for simple groups and, in particular, on the 2-local geometries for the sporadic groups. Part 2, consisting of two chapters, presents the main results of this book. Chapter 7 states the main results on homotopy decompositions for the 26 sporadic groups  $G$ . Proofs for primarily group-theoretic arguments are relegated to Chapter 8.

Reviewer: [Hirokazu Nishimura \(Tsukuba\)](#)

**MSC:**

- 20-02 Research monographs (group theory)
- 20D08 Simple groups: sporadic finite groups
- 20J06 Cohomology of groups
- 55-02 Research monographs (algebraic topology)
- 55R35 Classifying spaces of groups and  $H$ -spaces
- 20J05 Homological methods in group theory
- 55R40 Homology of classifying spaces, characteristic classes

Cited in <b>1</b> Review Cited in <b>10</b> Documents
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**Keywords:**

sporadic simple groups; classifying spaces; simplicial sets; Bousfield-Kan completions; homotopy colimits; homology decompositions; model categories; 2-local geometries