zbMATH
the first resource for mathematics

Marsden, Jerrold E.; **Misiołek, Gerard**; **Ortega, Juan-Pablo**; **Perlmutter, Matthew**; **Ratiu, Tudor S.**

Hamiltonian reduction by stages. (English) Zbl 1129.37001

Lecture Notes in Mathematics 1913. Berlin: Springer (ISBN 978-3-540-72469-8/pbk). xvi, 519 p. (2007).

This book is concerned with symplectic reduction by stages for Hamiltonian systems with symmetry. The book consists of three parts. The first part, consisting of three chapters, gives background from those parts of geometric mechanics that are needed in the remainder of the book. The first chapter contains background on regular symplectic reduction and includes all the proofs of the main theorems, such as point reduction, coadjoint orbits and orbit reduction. The second chapter begins with a review on connections on principal bundles, including curvature, which is needed for cotangent bundle reduction. The third chapter gives the problem setting and explains why reduction by stages is relatively routine in the Poisson setting, while being quite nontrivial in the symplectic setting.

The second part, consisting of 9 chapters, deals with the theory of Hamiltonian reduction by stages in the regular case. The second part begins with Chapter 4, which develops two of the basic results on reduction by stages, namely the case of commuting reduction and semidirect product reduction. These two special cases are important in applications as well as for the historical development of the subject. Chapter 5 develops the theory of reduction by stages in the case of free actions. A sufficient condition, called the stages hypothesis, under which the two step reduced space is symplectically diffeomorphic to the space obtained by reducing all at once by the original group, is stated. In Chapters 11 and 12 alternative conditions for reduction by stages are presented based on the use of distribution theory. The aim in Chapter 6 is to introduce hypotheses under which reduction by stages works. This will be followed by the actual reduction by stages procedure in Chapters 8, 9, and 10. Chapter 8 addresses the basic theory of symplectic reduction by stages for central extensions, which is followed by examples in Chapter 9. Chapter 10 deals with two themes. The first theme of the chapter is concerned with the semidirect product of a group with an abelian group. The second theme of the chapter is concerned with the semidirect product of two Lie groups, both of which can be nonabelian. Chapter 7 shows how to reduce cotangent bundles with magnetic terms.

The third part of the book develops the theory in the singular case. The techniques of this part are based on the theory of distributions, and they were introduced by *J.-P. Ortega* and *T. S. Ratiu* [Geometry, mechanics, and dynamics. Volume in honor of the 60th birthday of J. E. Marsden. New York, NY: Springer, 329–362 (2002; Zbl 1015.53053)] and *J.-P. Ortega* [Differ. Geom. Appl. 19, No. 1, 61–95 (2003; Zbl 1038.53076)]. This part consists of three chapters. The first chapter of this part gives a brief description of the optimal momentum map and explains how to construct symplectic or Marsden-Weinstein reduced spaces in the context in which this object is defined. The contents of this preliminary chapter is borrowed from Ortega and Ratiu (loc. cit.) and *J.-P. Ortega* [C. R., Math., Acad. Sci. Paris 334, No. 11, 999–1004 (2002; Zbl 1073.37065)]. The second chapter of this part investigates the orbit reduction procedure in the context of the optimal momentum map, where an orbit reduction is an approach to symplectic reduction equivalent to the Marsden-Weinstein reduction used in the second part of the book. The final chapter of the third part of the book extends to the optimal context the reduction by stages procedure studied in the second part for the standard momentum map.

The book focuses on symplectic reduction by stages motivated by both applications and the intrinsic mathematical structure, and does not deal with Poisson reduction, Lagrangian reduction, Routh reduction, etc. The book does not cover the analytical theory in the infinite-dimensional case either, though many of the most interesting examples are in fact infinite-dimensional. The book does not cover the interesting links of reduction theory with representation theory, symplectic topology and quantization. Of course, it would be absurd to believe that any book of reasonable size can cover all these topics.

Reviewer: Hirokazu Nishimura (Tsukuba)

Edited by FIZ Karlsruhe, the European Mathematical Society and the Heidelberg Academy of Sciences and Humanities
© 2019 FIZ Karlsruhe GmbH

MSC:

37-02	Research exposition (dynamical systems and ergodic theory)
37J15	Symmetries, invariants, invariant manifolds, momentum maps, reduction
53D20	Momentum maps; symplectic reduction
70H03	Lagrange's equations
70H05	Hamilton's equations
70H33	Symmetries and conservation laws, reverse symmetries, invariant manifolds, etc.

Cited in **1** Review
Cited in **48** Documents

Keywords:

symplectic reduction; Hamiltonian system; semidirect product reduction; commuting reduction; the stages hypothesis; optimal momentum map; distribution; orbit reduction

Full Text: DOI

© 2019 FIZ Karlsruhe GmbH