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Pseudo limits, biadjoints, and pseudo algebras: categorical foundations of conformal field theory. (English) Zbl 1100.81036

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Conformal field theory is of paramount importance to both physicists and mathematicians. lt is so to physicists, because it yields one approach to string theory, which is intended to unify the four fundamental forces of nature. It Is so to mathematicians, because it provides a geometric definition of elliptic cohomology, which is related to *R. E. Borcherd*'s proof [Invent. Math. 109, No. 2, 405–444 (1992; Zbl 0799.17014)] of the Moonshine conjecture.

The aim of this long paper is to develop the categorical foundations needed for working the rigorous approach to the definition of conformal field theory outlined by Segal [*R. Street*, J. Pure Appl. Algebra 2, No. 2, 149–168 (1972; Zbl 0241.18003)]. The author introduces the general concepts of weighted bilimits, weighted bicolimits, and biadjoints for pseudo functions between 2-categories, and proves statements about their existence in certain cases. Although these were discussed in [(*) Commun. Math. Phys. 254, No. 1, 221–253 (2005; Zbl 1091.81073)], the terminology there was not loyal to the established categorical one. What was called a lax algebra in (*) should have been called a pseudo algebra. What was called a lax morphism in (*) should have been called a pseudo morphism or simply a morphism. What was called a lax functor in (*) should have been called a pseudo functor. This inspired the present author to give a precise translation of the notions in (*). There are many different ways to weaken 1-categorical notions. To mention a few [*J. W. Gray*, Formal category theory: adjointness for 2-categories, Springer-Verlag, Berlin, 1974, Lecture Notes Math. 391, Berlin: Springer (1974; Zbl 0285.18006), Bull. Am. Math. Soc. 80, 142–147 (1974; Zbl 0358.18005), Bull. Aust. Math. Soc. 39, No. 2, 301–317 (1989; Zbl 0657.18004) and *A. Vistoli*, Notes on Grothendieck topologies, fibered categories and descent theory]. The author sets up the weakened notions for the unique purpose of utilizing stacks to rigorously define conformal field theory as by *P. Hu* and *I. Križ* [Adv. Math. 189, No. 2, 325–412 (2004; Zbl 1071.55004)]. The circumstances of conformal field theory strongly suggest a particular choice of concepts. The structure present an the class C of rigged surfaces is captured by these concepts of 2-category theory. Concepts of 2-categories enter when we describe the operations of disjoint union of two rigged surfaces and gluing along boundary components of opposite orientation. We may say that C as well as C(M) and QM,H) from page 235 of *P. Hu* and *I. Križ* [Commun. Math. Phys. 254, No. 1, 221–253 (2005; Zbl 1091.81073)] is a stack of pseudo commutative monolds with cancellation. The theory and 2-theory apparatus gives us a concise way to List necessary coherence isos and coherence diagrams. This paper gives a thorough treatment of theories, 2-theories, their pseudo algebras, and their relevant diagrams.

The main results of the paper are as follows:

It is shown constructively that every pseudo functor from a 1-category to the 2-category of small categories admits both a pseudo limit and a pseudo colimit. Besides, the 2-category of small categories admits weighted pseudo limits and weighted pseudo colimits It is also shown, by an adaption of the proof for small categories, that any pseudo functor from a 1-category to the 2-category of pseudo T-algebras admits a pseudo limit. After a proof of the existence of cotensor products in the 2 category of pseudo T-algebras, the author concludes from a theorem of Street that this 2 category admits weighted pseudo limits. Then the author turns to biadjoint. It is shown in analogy with the corresponding result in 1-category theory that a pseudo functor admits a left biadjoint ideal we have an appropriate biuniversal arrow for each object of the source category. This enables the author to show that the forgetful functor from the 2-category of pseudo T-algebras to that of pseudo S-algebras associated with any morphism of theories $\phi : S \to T$ admits a left biadjoint. After proving that this 2-category admits both bicolimits and bitensor products, the author concludes that it also admits weighted bicolimits. Finally, the author constructs pseudo limits of pseudo algebras over a 2-theory. Using a theorem of Street and the existence of cotensor products, the author concludes that the 2-category of pseudo algebras over a 2-theory admits weighted pseudo limits.

The paper consists of 13 chapters. Chapters 3, 4, 6, 11 and 13 are devoted to the existence of various types of limits in various 2-categories. In particular, the fundamentals of Lawvere theories and algebras are treated in Chapter 6. The passage from strict algebras to pseudo algebras is discussed in Chapter 7. Chapters 9 and 10 deal with biadjoints, which allow a universal description of the stack of covering spaces an page 337 of Hu-Križ (2004). Grothendieck topologies and stacks are discussed in Chapter 12. The 2-theory of commutative monolds with cancellation along with the example of rigged surfaces is presented in Chapter 3.

This paper will skillfully inoculate the reader with what the author intended to get across, and it will be long time value to both conformal field theorists who want to master 2-category theory and category theorists who want to know the 2-categorical structure of conformal field theory.

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MSC:

81T40 Two-dimensional field theories, conformal field theories, etc.
18C10 Theories, structure, and semantics
18C20 Algebras and Kleisli categories associated with monads
18A30 Limits; colimits

Keywords:

pseudo algebra; stack; 2-category theory

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