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Renormalization and effective field theory. (English) Zbl 1221.81004

Mathematical Surveys and Monographs 170. Providence, RI: American Mathematical Society (AMS) (ISBN 978-0-8218-5288-0/hbk). vii, 251 p. (2011).

Most axiomatic formulations of quantum field theory in the literature start from the Hamiltonian formulation of the theory. This book proposes a novel axiomatic formulation of the theory based on the belief that the Lagrangian formulation of quantum field theory, using Feynman's sum over histories, is more fundamental. The basic idea is very simple. The part of physics that occurs below a certain energy, say Λ, is succinctly encoded by the Lagrangian $S^{\mathrm{eff}}[\Lambda]$. Note that the notorious infinities of quantum field theory occur only if we take physical phenomena of arbitrarily high energy into consideration. A continuum quantum field theory is merely a sequence of low-energy effective actions $S^{\mathrm{eff}}[\Lambda]$ for all Λ subject to the locality axiom that interactions between fields occur at points, as energy scales go to infinity.

Consisting of 6 chapters and two appendices, the book aims to develop complete foundations for perturbative quantum field theory in Riemannian signature on any manifold under the above direction. All the main results of this book are based upon the existence theorem established in Chapter 2. It claims, roughly speaking, that there are quantum field theories wherever there are Lagrangians. Chapter 3 shows how the main theorem of Chapter 2 works on $\mathbb{R}^n$. Chapter 4 is devoted to renormalizability. The basic philosophy of the book is that the counterterms themselves have no intrinsic importance. They should be regarded as a mere tool in the construction of the bijection between theories and local action functionals. The author classifies all possible renormalizable scalar field theories, and finds the expected result. In Chapter 5 the author discusses how to incorporate gauge theories into his formalism. The author's standpoint is that everything one says about a quantum field theory should be said in terms of its effective theories. The bijection between theories and local action functionals allows one to translate statements about effective theories into those about local action functionals. Naively speaking, to give a gauge theory would be to give an effective gauge theory at every energy level in a way related by the renormalization group flow. The problem with this idea is that the space of low-energy gauge symmetries is not a group, namely, the product of low-energy symmetries is no longer low-energy. This obstacle is overcome by the synthesis of the Wilsonian effective action picture and the Batalin-Vilkovisky formalism. The author develops a cohomological approach to construct theories which are renormalizable and which satisfy the quantum master equation. It is proved that, given any classical gauge theory subject to the classical analog of renormalizability, one can construct its renormalizable quantization with a certain cohomology group vanishing. In Chapter 6 the author applies this general theorem so as to establish the renormalizability of pure Yang-Mills theories, for which he is forced to calculate the Gel'fand-Fuchs cohomology groups controlling obstructions and deformations. The result is not so satisfactory, because the success is brought out by the fortuitous vanishing of the cohomology groups, namely,

$$H^5\big(\mathfrak{su}(3)\big)^{\mathrm{Out}(\mathfrak{su}(3))} = 0.$$

Two appendices on asymptotics of graph integrals and nuclear spaces are accompanied.

In the last decade or so, the perturbative version of algebraic quantum field theory has been investigated in [*R. Brunetti* and *K. Fredenhagen*, Commun. Math. Phys. 208, No. 3, 623–661 (2000; Zbl 1040.81067), Lect. Notes Phys. 786, 129–155 (2009; Zbl 1184.81099); *M. Dütsch* and *K. Fredenhagen*, Commun. Math. Phys. 219, No. 1, 5–30 (2001; Zbl 1019.81041); *S. Hollands* and *R. M. Wald*, Commun. Math. Phys. 293, No. 1, 85–125 (2010; Zbl 1193.81076)] and so on, where results similar to those in the book are established. These results depend on a version of the Epstein-Glaser construction of counterterms [*H. Epstein* and *V. Glaser*, Ann. Inst. Henri Poincaré, Nouv. Sér., Sect. A 19(1973), 211–295 (1974; Zbl 1216.81075)]. This book, as well as those works, constructs counterterms directly on real space, in sharp contrast with the construction on momentum-space, where complicated graph combinatorics is inevitable. Another related approach to perturbative quantum field theory on Riemannian space-times was developed by *S. Hollands* [SIGMA, Symmetry Integrability Geom. Methods Appl. 5, Paper 090, 45 p. (2009; Zbl 1188.81127)] and *S. Hollands* and *H. Olbermann* [J. Math. Phys. 50, No. 11, Paper No. 112304, 42 p. (2009; Zbl 1304.81113)], where the field theory is encoded in a vertex algebra on the space-time manifold.

I conclude this review with an intriguing question. An approach to perturbative renormalization has been developed also in [*A. Connes* and *D. Kreimer*, NATO ASI Ser., Ser. C, Math. Phys. Sci. 530, 59–108 (1999; Zbl 1041.81086); *A. Connes* and *M. Marcolli*, Int. Math. Res. Not. 2004, No. 76, 4073–4091 (2004; Zbl 1131.81021); *W. D. van Suijlekom*, Commun. Math. Phys. 276, No. 3, 773–798 (2007; Zbl 1194.81165); *A. Connes* and *D. Kreimer*, Commun. Math. Phys. 210, No. 1, 249–273 (2000; Zbl 1032.81026), Commun. Math. Phys. 216, No. 1, 215–241 (2001; Zbl 1042.81059); *A. Connes* and *M. Marcolli*, Noncommutative geometry, quantum fields and motives. Colloquium Publications. American Mathematical Society 55. Providence, RI: American Mathematical Society (AMS)/Hindustan Book Agency (2008; Zbl 1209.58007)] and so on. The first result of this approach is that the Bogoliubov-Parasiuk-Hepp-Zimmermann algorithm is of a beautiful interpretation in terms of the Birkhoff decomposition for loops in a certain pro-algebraic group constructed combinatorially from graphs, while counterterms are of no intrinsic importance in this book. It remains obscure whether there is any relationship between Connes-Kreimer's Hopf algebras and the results of this book.

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

81-02	Research monographs (quantum theory)
81Txx	Quantum field theory and related classical field theories
14N35	Gromov-Witten invariants, quantum cohomology, etc.

Keywords:

renormalization; quantum field theory; gauge theory; counterterm; Lagrangian; axiomatic quantum field theory; perturbative quantum field theory; locality axiom; statistical field theory; Wilsonian low energy theory; local action functional; Gel'fand-Fuchs cohomology; continuum quantum field theory; renormalization group flow; effective quantum field theory