Kohno, Toshitake

Geometry of iterated integrals. (Hanpuku sekibun no kikagaku). (Japanese) Zbl 1305.26004

Tokyo: Springer (ISBN 978-4-431-70669-4). vii, 295 p. (2009).

Chen's iterated integrals were introduced and investigated in [Zbl 0217.47705], [Zbl 0229.58002], [Zbl 0227.58003], [Zbl 0271.58001], [Zbl 0301.58006], [Zbl 0345.58003], [Zbl 0389.58001], [Zbl 0409.58013] and [Zbl 0453.58004] in the 1970's. As far as the reviewer knows, this book is peerless among countless mathematical books, whatever languages they may be written in, in that it is devoted exclusively to Chen's iterated integrals. The reviewer urges vehemently that the book should be translated into English.

Quillen's rational homotopy theory in [Zbl 0191.53702] has established a systematic method in computing higher homotopy groups while neglecting torsion, and Chen's iterated integrals could be regarded as a way of computing rational homotopy in terms of differential forms. As the de Rham cohomology is innocent of torsion, Chen's theory of iterated integrals could be called the de Rham homotopy theory. Chen's theory lies at the entrance to the infinite-dimensional differential geometry of loop spaces, for it deals with a finitary portion of the whole infinite-dimensional geometry.

The book consists of seven chapters. The first chapter explains iterated integrals of differential 1-forms with only freshman or sophomore calculus as prerequisites, discussing their applications to total differential equations on $\mathbb{R}^n$ and multiple logarithmic functions.

From the second chapter on, the reader is assumed to be acquainted modestly with the theory of smooth manifolds. The second chapter formulates iterated integrals generally and introduces the bar complex $\mathcal{B}^*(M)$ on the loop space ΩM of a smooth manifold M.

The main result of Chapter 3 is that the cohomology of $\mathcal{B}^*(M)$ is isomorphic to the cohomology group $H^*(\Omega M;\mathbb{R})$ provided M is a simply connected smooth manifold ([Zbl 0227.58003]). The proof relies on Adams' cobar construction ([Zbl 0071.16404] and [Zbl 0086.37502]). The description of the de Rham cohomology on free loop spaces in terms of iterated integrals is also discussed. The fourth chapter is devoted to formal homology connections and their holonomies discussed in [Zbl 0389.58001], by which the algebraic structure of the homology of the loop space ΩM is elucidated. The fifth section of the chapter deals with the homology of ΩM with integer coefficients with applications to Hopf invariants of mappings between spheres. The final section of the chapter touches upon string topology, which is fully discussed in [Zbl 1089.57002].

The fifth chapter is concerned with the main result of [Zbl 0345.58003]. The concluding two sections (§5.6 and §5.7) of this chapter are devoted to the relationship between Sullivan's minimal models in [Zbl 0374.57002] and the method discussed in the previous chapter, which stand to each other in duality and give essentially the same information. A full treatment of rational homotopy in terms of minimal models can be seen in [Zbl 0961.55002].

The sixth chapter begins with hyperplane arrangements (§6.1), which is based largely upon the author's [Zbl 0995.32018]. Generally speaking, so-called Riemann-Hilbert problems are a type of problems in which one is required, given a representation of the fundamental group, to construct an integrable connection of the representation as its monodromy representation. The second section of this chapter investigates Riemann-Hilbert problems with unipotent representations of the fundamental group. The main result of this section is to be attributed to [Zbl 0416.32020]. The third section of this chapter discusses finite-type invariants for braid groups, which is based largely upon the author's [Zbl 0978.57006]. The fourth section of this chaper discusses Kontsevich integrals ([Zbl 0839.57006]). The fifth section of this chapter deals with Drinfel'd associators ([Zbl 0718.16003]). The final section (§6.6) describes the algebraic structure of the homology of the loop space $\Omega\mathcal{C}_n(\mathbb{R}^m)$ with $m \geq 3$, where $\mathcal{C}_n(\mathbb{R}^m)$ stands for the configuration space, namely,

$$\mathcal{C}_n(\mathbb{R}^m) = (\mathbb{R}^m)^n \setminus \bigcup_{i<j} \Delta_{ij}$$

with

$$\Delta_{ij} = \{(\mathbf{x}_1, \dots, \mathbf{x}_n) \in (\mathbb{R}^m)^n \mid \mathbf{x}_i = \mathbf{x}_j\}$$

Chapter 7, which is relatively independent of the previous chapters, discusses the method in [Zbl 0382.33010] and its applications.

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

Cited in **2** Documents

26-01	Textbooks (real functions)
26B20	Integral formulas (Stokes, Gauss, Green, etc.)
28A25	Integration with respect to measures and other set functions
49Q15	Geometric measure and integration theory, integral and normal currents (optimization)

Keywords:

iterated integral; Kontsevich integral; Vassiliev invariant; configuration space; loop space; de Rham homotopy theory; minimal model; fundamental group; monodromy; Riemann-Hilbert problem; hyperplane arrangement; bar complex; formal homology connection; Lobachevsky function; Schläfli equality; spherical orthosimplex; hyperbolic geometry; spherical geometry; braid; Adams' cobar construction; configuration space; Orlik-Solomon algebra; chord diagram; multiple zeta function; rational homotopy theory; Hopf algebra