

Iglesias-Zemmour, PatrickThe moment maps in diffeology. (English) [Zbl 1203.53084](#)

Mem. Am. Math. Soc. 972, v, 72 p. (2010).

It was *J. Souriau* [Structure des systèmes dynamiques. Maitresses de mathématique. Paris: Dunod (1970; [Zbl 0186.58001](#))] who introduced the notion of a moment map for the first time. The tool deals with symmetries in symplectic or pre-symplectic geometry. The author of this paper, who was once a student of Souriau, is most enthusiastic in developing diffeology, the ideas of which date back to [*J. M. Souriau*, Lect. Notes Math. 836, 91–128 (1980; [Zbl 0501.58010](#)); *K.-T. Chen*, Bull. Am. Math. Soc. 83, 831–879 (1977; [Zbl 0389.58001](#)); Trans. Am. Math. Soc. 206, 83–98 (1975; [Zbl 0301.58006](#)); Ann. Math. (2) 97, 217–246 (1973; [Zbl 0227.58003](#))]. He has already written a monograph on diffeology [Diffeology. Mathematical Surveys and Monographs 185. Providence, RI: American Mathematical Society (2013; [Zbl 1269.53003](#))]. The principal objective in this memoir is to extend the notion of symplectic formalism and moment maps to diffeology. As the author has stated, “the moment map is just an object of the world of differential closed form, and there is no reason a priori that it could not be extended to diffeology which has a very well developed framework for De Rham’s calculus.”

Given a diffeological space X equipped with a closed 2-form ω and a diffeological group G acting on M with preserving ω , the space \mathcal{G}^* of left-invariant 1-forms is called the space of momenta. The difficulty is that not every G -invariant closed form is exact, and that there is no reason to assume its primitives, if any, to be G -invariant. To overcome this difficulty, the author considers the space Paths(X) of all smooth paths on X , on which we get a differential 1-form by integrating ω along the paths. The moment map Ψ on Paths(X) is then put down to the moment map μ on X taking its values in some quotient of the space of moments in place of the space of moments itself. By taking G to be the group Diff(X, ω) of all diffeomorphisms of X preserving ω , we get a universal one. The last chapter is devoted to several examples involving diffeological groups or diffeological spaces essentially. The author’s approach reveals the fact that the theory of moment maps should proceed covariantly, avoiding such contravariant objects as Lie algebras or vector fields.

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

- 53D20 Momentum maps; symplectic reduction
53C99 Global differential geometry
53D30 Symplectic structures of moduli spaces
58A03 Topos-theoretic approach to differentiable manifolds

Cited in 2 Documents

Keywords:

diffeology; moment map; symplectic geometry

Full Text: DOI**References:**

- [1] Augustin Banyaga, Sur la structure du groupe des difféomorphismes qui préservent une forme symplectique, Comment. Math. Helv. 53 (1978), no. 2, 174–227 (French). · [Zbl 0393.58007](#) · [doi:10.1007/BF02566074](#)
- [2] William M. Boothby, Transitivity of the automorphisms of certain geometric structures, Trans. Amer. Math. Soc. 137 (1969), 93–100. · [Zbl 0181.49503](#) · [doi:10.1090/S0002-9947-1969-0236961-0](#)
- [3] Raoul Bott, On some formulas for the characteristic classes of group-actions, Differential topology, foliations and Gelfand-Fuks cohomology (Proc. Sympos., Pontifícia Univ. Católica, Rio de Janeiro, 1976) Lecture Notes in Math., vol. 652, Springer, Berlin, 1978, pp. 25–61. · [Zbl 0096.17701](#)
- [4] M. Condevaux, P. Dazord, and P. Molino, Géométrie du moment, Travaux du Séminaire Sud-Rhodanien de Géométrie, I, Publ. Dép. Math. Nouvelle Sér. B, vol. 88, Univ. Claude-Bernard, Lyon, 1988, pp. 131–160 (French).
- [5] Kuo Tsai Chen, Iterated path integrals, Bull. Amer. Math. Soc. 83 (1977), no. 5, 831–879. · [Zbl 0389.58001](#) · [Zbl 0389.58001](#)

[doi:10.1090/S0002-9904-1977-14320-6](https://doi.org/10.1090/S0002-9904-1977-14320-6)

- [6] S. K. Donaldson, Moment maps and diffeomorphisms, *Asian J. Math.* 3 (1999), no. 1, 1-15. Sir Michael Atiyah: a great mathematician of the twentieth century. · [Zbl 0999.53053](#) · [doi:10.4310/AJM.1999.v3.n1.a1](#)
- [7] Paul Donato Revêtement et groupe fondamental des espaces différentiels homogènes Thèse de doctorat d'état, Université de Provence, Marseille, 1984.
- [8] Paul Donato, Géométrie des orbites coadjointes des groupes de difféomorphismes, *Géométrie symplectique et mécanique* (La Grande Motte, 1988), Lecture Notes in Math., vol. 1416, Springer, Berlin, 1990, pp. 84-104 (French). · [doi:10.1007/BFb0097466](#)
- [9] Patrick Iglesias Fibrés difféologiques et homotopie Thèse de doctorat d'état, Université de Provence, Marseille, 1985. [http://math.huji.ac.il/\(\sim\)pix/documents/These.pdf](http://math.huji.ac.il/(\sim)pix/documents/These.pdf)
- [10] Patrick Iglesias, Yael Karshon and Moshe Zadka Orbifolds as diffeologies 2005 <http://arxiv.org/abs/math.DG/0501093>
- [11] P. Iglesias and G. Lachaud, Espaces différentiables singuliers et corps de nombres algébriques, *Ann. Inst. Fourier (Grenoble)* 40 (1990), no. 3, 723-737 (French, with English summary). · [Zbl 0703.57017](#)
- [12] Patrick Iglesias, La trilogie du moment, *Ann. Inst. Fourier (Grenoble)* 45 (1995), no. 3, 825-857 (French, with English and French summaries). · [Zbl 0836.58001](#)
- [13] Patrick Iglesias-Zemmour Diffeology eprint 2005-07 [http://math.huji.ac.il/\(\sim\)pix/diffeology/](http://math.huji.ac.il/(\sim)pix/diffeology/)
- [14] Patrick Iglesias-Zemmour, Diffeology of the infinite Hopf fibration, *Geometry and topology of manifolds*, Banach Center Publ., vol. 76, Polish Acad. Sci. Inst. Math., Warsaw, 2007, pp. 349-393. · [Zbl 1115.58009](#) · [doi:10.4064/bc76-0-17](#)
- [15] Patrick Iglesias-Zemmour, Dimension in diffeology, *Indag. Math. (N.S.)* 18 (2007), no. 4, 555-560. · [Zbl 1144.58006](#) · [doi:10.1016/S0019-3577\(07\)80062-0](#)
- [16] Patrick Iglesias-Zemmour Variations of integrals in diffeology eprint 2007 [http://math.huji.ac.il/\(\sim\)pix/documents/VOIID.pdf](http://math.huji.ac.il/(\sim)pix/documents/VOIID.pdf)
- [17] Patrick Iglesias-Zemmour. Every symplectic manifold is a coadjoint orbit eprint 2007 [http://math.huji.ac.il/\(\sim\)pix/documents/ESMI](http://math.huji.ac.il/(\sim)pix/documents/ESMI)
- [18] A. Kirillov, Éléments de la théorie des représentations, Éditions Mir, Moscow, 1974 (French). Traduit du russe par A. Sossinsky [A. B. Sosinskij].
- [19] Bertram Kostant, Orbits and quantization theory, *Actes du Congrès International des Mathématiciens* (Nice, 1970) Gauthier-Villars, Paris, 1971, pp. 395-400. · [Zbl 0233.22006](#)
- [20] Stephen M. Omohundro, Geometric perturbation theory in physics, World Scientific Publishing Co., Singapore, 1986. · [Zbl 0830.58036](#)
- [21] J.-M. Souriau, Structure des systèmes dynamiques, Maçflexitises de mathématiques, Dunod, Paris, 1970 (French). · [Zbl 0186.58001](#)
- [22] J.-M. Souriau, Groupes différentiels, Differential geometrical methods in mathematical physics (Proc. Conf., Aix-en-Provence/Salamanca, 1979) Lecture Notes in Math., vol. 836, Springer, Berlin-New York, 1980, pp. 91-128 (French).
- [23] Jean-Marie Souriau Groupes différentiels de physique mathématique Lecture Notes in Physics Springer Verlag Berlin - Heidelberg 201 511 - 513 1984
- [24] François Ziegler Théorie de Mackey symplectique, in Méthode des orbites et représentations quantiques Thèse de doctorat d'Université, Université de Provence, Marseille, 1996.

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.