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Directed algebraic topology. Models of non-reversible worlds. (English) Zbl 1176.55001

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Directed algebraic topology should be distinguished from classical algebraic topology by the principle that directed spaces have privileged directions and directed paths therein need not be reversible. The main structures for directed algebraic topology are topological spaces with directed paths closed under constant paths, partial increasing reparametrizations and concatenation.

Such spaces, together with continuous mappings preserving directed paths, form a category **dTop** containing a "directed circle" $\uparrow \mathbf{S}^1$ and higher directed spheres $\uparrow \mathbf{S}^n$, all of which arise from the discrete two-point space under directed suspension. Applications of directed algebraic topology can be found in such areas as concurrent processes, rewriting systems, traffic networks, space-time models, biological systems, etc., where privileged directions occur naturally. The most developed applications are concerned with concurrency, for which the reader is referred to [*L. Fajstrup, M. Raussen* and *E. Goubault*, Theor. Comput. Sci. 357, No. 1–3, 241–278 (2006; Zbl 1099.55003); *L. Fajstrup*, European women in mathematics. Proceedings of the 9th general meeting (EWM'99), Loccum, Germany, August 30 – September 4, 1999. Stony Brook, NY: Hindawi Publishing Corporation. 135–140 (2000; Zbl 1012.68076)], etc.

The author is keenly aware of the relationship between **dTop** and the category **Cub** of cubical sets. The latter enjoys privileged directions in any dimension, while the former is essentially based on 1-dimensional information. Although elementary paths and homotopies can not be concatenated in **Cub**, higher homotopy properties of **Cub** can be studied with the geometric realization functor **Cub** $\to$ **dTop** and the notion of relative equivalence it provides.

The abstract cylinder functor I and natural transformations between its powers, like faces, degeneracy, connections, and the like play a fundamental role. What is crucial, such formal structures are "categorically algebraic" in much the same way as the theory of monads. In the classical case, settings based on the cylinder endofunctor are traced back to *D. M. Kan*'s famous series of papers on "abstract homotopy" [Proc. Natl. Acad. Sci. USA 41, 1092–1096 (1955; Zbl 0065.38601), ibid. 42, 255–258 (1956; Zbl 0071.16702), ibid. 42, 419–421 (1956; Zbl 0071.16801) and ibid. 42, 542–544 (1956; Zbl 0071.16901)], and *K. H. Kamps* and *T. Porter*'s book [Abstract homotopy and simple homotopy theory. Singapore: World Scientific (1997; Zbl 0890.55014)] is a general reference on such settings.

Directed algebraic topology is of much help in providing a sort of geometric intuition for category theory, in a sharper way than classical algebraic topology which can provide intuition for the theory of groupoids, a reversible version of categories. The relationship between directed algebraic topology and category theory is even stronger in higher dimensions, which is still under research, so that the reader is only referred to [*M. Grandis*, Homology Homotopy Appl. 8, No. 1, 31–70 (2006; Zbl 1087.18005), Appl. Categ. Struct. 14, No. 3, 191–214 (2006; Zbl 1103.18008) and Cah. Topol. Géom. Différ. Catég. 47, No. 2, 107–128 (2006; Zbl 1170.18301)] and the references therein.

The book consists of six chapters with an appendix on category theory. The six chapters are divided evenly into two parts, the first part dealing with first-order directed homotopy and homology while the second part is concerned with higher-order homotopy theory. Basic settings are covered in Chapter 1, while Chapter 4, based mainly on the author's work in [Rend. Ist. Mat. Univ. Trieste 25, No. 1–2, 223–262 (1993; Zbl 0834.55012), Ann. Mat. Pura Appl., IV. Ser. 170, 147–186 (1996; Zbl 0871.55016) and Appl. Categ. Struct. 5, No. 4, 363–413 (1997; Zbl 0906.55015)], is devoted to a complete reworking of these settings, which is aimed mostly at the reversible case. Chapter 2 studies the directed homology of cubical sets. The study of the fundamental category of a directed space, via minimal models up to directed homotopy of categories, is developed in Chapter 3. Chapter 5 is concerned with categories of diagrams and sheaves, slice categories, categories of algebras for a monad and categories of differential graded algebras. Chapter 6 is the study on cases where paths have a weight or cost expressing length, price, energy, etc.

Directed algebraic topology is a relatively new field that arose in the 1990s, and this first book on the subject will skillfully lead the reader to its essentials.

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

55-01	Textbooks (algebraic topology)
55U35	Abstract homotopy theory; axiomatic homotopy theory
18D05	Double categories, 2-categories, bicategories and generalizations
68Q85	Models and methods for concurrent and distributed computing

Keywords:

directed algebraic topology; cubical sets; abstract homotopy theory; concurrency; category theory; cylinder functor; groupoid; reversor; weighted algebraic topology; traffic network