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Toric varieties. (English) [\[Zbl 1223.14001\]](#)

Graduate Studies in Mathematics 124. Providence, RI: American Mathematical Society (AMS) (ISBN 978-0-8218-4819-7/hbk). xxiv, 841 p. (2011).

The study of toric varieties is a highly interesting part of algebraic geometry, having deep connections with polytopes, polyhedra, combinatorics, commutative algebra, symplectic geometry and topology while being of unexpected applications in such diverse areas as physics, coding theory, algebraic statistics and geometric modeling. The concreteness of toric varieties enables one to grasp the meaning of the powerful techniques of modern algebraic geometry firmly, providing a fertile testing ground for general theories.

This book is a good introduction to this rich subject, providing more details with numerous examples, figures and exercises than existing books [*G. Ewald*, Combinatorial convexity and algebraic geometry. Graduate Texts in Mathematics. 168. New York, NY: Springer (1996; [Zbl 0869.52001](#)); *W. Fulton*, Introduction to toric varieties. Annals of Mathematics Studies. 131. Princeton, NJ: Princeton University Press (1993; [Zbl 0813.14039](#)); *T. Oda*, Convex bodies and algebraic geometry. An introduction to the theory of toric varieties. Ergebnisse der Mathematik und ihrer Grenzgebiete. 3. Folge, Bd. 15. Berlin etc.: Springer-Verlag (1988; [Zbl 0628.52002](#))].

The book, consisting of 15 chapters, begins with affine toric varieties in Chapter 1 and projective toric varieties in Chapter 2. Chapter 3 is concerned with normal varieties, though the definition of variety does not assume normalcy. Chapter 4 considers Weil divisors and Cartier divisors, which coincide on a smooth variety, but whose relationship is more complicated for a normal variety. It is shown that normal varieties are the natural setting to develop a theory of divisors and divisor classes. Chapter 5 demonstrates that the classical construction \mathbb{P}^n can be generalized to any toric variety X_Σ . Chapter 6 relates Cartier divisors to invertible sheaves on X_Σ . The structure of the nef cone and its dual called the Mori cone is described in detail. In Chapter 7 the authors extend the relation between polytopes and projective toric varieties to that between polyhedra and projective toric morphisms $\phi : X_\Sigma \rightarrow U_\Sigma$. Projective bundles over a toric variety are discussed, so that smooth projective toric varieties of Picard number 2 are classified. In Chapter 8 Weil divisors are related to reflexive sheaves of rank one, where Zariski p -forms are defined. Chapter 9 is devoted to sheaf cohomology. Chapter 10 is concerned with the structure of 2-dimensional normal toric varieties (toric surfaces). Their singularities are described, and smooth complete toric surfaces are classified. Chapter 11 establishes the existence of toric resolutions of singularities for toric varieties of all dimensions. The goal in Chapter 12 is to understand some topological invariants of a toric variety X with applications to polytopes. Chapter 13 proves the Hirzebruch-Riemann-Roch theorem for a line bundle $\mathcal{O}_X(D)$ on a smooth complete toric variety X_Σ . Chapters 14 and 15 study the GIT (Geometric Invariant Theory) quotients $\mathbb{C}^r //_\chi G$ as $\chi \in \widehat{G}$ varies. The full story of what happens as χ varies is controlled by the secondary fan, which is the main topic of the last section of Chapter 14. The aim in Chapter 15 is to understand the structure of the GKZ (Gel'fand, Kapranov and Zelevinsky) cones and what happens to the associated toric varieties as one moves around the secondary fan. The book is accompanied by three appendices on the history of toric varieties, computational methods and spectral sequences.

Reviewer: [Hirokazu Nishimura \(Tsukuba\)](#)

MSC:

- [14-01](#) Textbooks (algebraic geometry)
- [14M25](#) Toric varieties, Newton polyhedra
- [14C17](#) Intersection theory, etc.
- [14C15](#) (Equivariant) Chow groups and rings; motives

Cited in **4** Reviews
Cited in **314** Documents

Keywords:

[toric variety](#); [Weil divisor](#); [Cartier divisor](#); [convexity](#); [Picard number](#); [polytope](#); [polyhedron](#); [sheaf cohomology](#); [Hirzebruch-Jung continued fraction](#); [Gröbner fan](#); [McKay correspondence](#); [Rees algebra](#); [multiplier ideal](#); [Hirzebruch-Riemann-Roch theorem](#); [Chow ring](#); [intersection cohomology](#); [invariant theory](#);

GKZ cone; secondary fan; spectral sequence