Marcolli, Matilde

Feynman motives. (English) Zbl 1192.14001
Hackensack, NJ: World Scientific (ISBN 978-981-4271-20-2/hbk; 978-981-4304-48-1/pbk; 978-981-4271-21-9/ebook). xiii, 220 p. (2010).

This book is based on the author's notes for the course on "Geometry and Arithmetic of Quantum Fields", which was taught at the California Institute of Technology in the fall of 2008.

There are two complementary approaches to understanding the relation between Feynman integrals and motives, namely, bottom-up and top-down approaches. The former approach looks at individual Feynman integrals for given Feynman graphs and identifies directly the Feynman integral with an integral of an algebraic differential form on a cycle in an algebraic variety by using the parametric representation in terms of Schwinger and Feynman parameters. It tries to understand the motivic nature of the piece of the relative cohomology of the algebraic variety involved in the computation of the period, trying to identify specific conditions under which it will be a realization of a very special kind of motive, a mixed Tate motive. The top-down approach is based on the formal properties that the category of mixed Tate motives satisfies, which are sufficiently rigid to identify it with a category of representations of an affine group scheme. This approach addresses the question of the relation to Feynman integrals by showing that the data of Feynman integrals for all graphs and arbitrary scalar field theories also fit together to form a category with the same properties.

The top-down approach is the focus of the author's joint work with Alain Connes on renormalization and the Riemann-Hilbert correspondence, and is presented in [Noncommutative geometry, quantum fields and motives. Texts and Readings in Mathematics 47. New Delhi: Hindustan Book Agency (2008; Zbl 1159.58004)] at some length. The bottom-up approach originated with the joint work of *S. Bloch, H. Esnault* and *D. Kreimer* [Commun. Math. Phys. 267, No. 1, 181–225 (2006; Zbl 1109.81059)], but the author prefers to refer to her joint work with Paolo Aluffi.

The book consists of 9 Chapters. The first Chapter gives a very brief introduction to perturbative quantum field theory, while the second Chapter gives a sketchy account of the theory of motives of algebraic varieties. Then the third Chapter begins to connect the two topics via the bottom-up approach. Following the author's preprint "Motivic renormalization and singularities" [`arXiv:0804.4824`], the fourth Chapter focuses on the similarities between the parametric form of Feynman integrals and the oscillatory integrals used in singularity theory.

It is well known in quantum field theory that the process of removing divergences from Feynman integrals via renormalization cannot be achieved without taking into account the nested structure of divergences. A process of subtracting divergences that takes into account this hierarchy of nested subdivergences was developed in [*N. N. Bogoliubow* and *O. S. Parasiuk*, Acta Math. 97, 227–266 (1957; Zbl 0081.43302)] and later completed in [Commun. Math. Phys., 2, 301–326 (1966)] and [*W. Zimmermann*, Commun. Math. Phys. 15, 208–234 (1969; Zbl 0192.61203)]. A vert elegant geometric formulation of this renormalization procedure was given in [*A. Connes* and *D. Kreimer*, Commun. Math. Phys. 210, No. 1, 249–273 (2000; Zbl 1032.81026) and Commun. Math. Phys. 216, No. 1, 215–241 (2001; Zbl 1042.81059)] in terms of a Hopf algebra of Feynman graphs. Chapter 5 gives a very brief account of the main steps in the Connes-Kreimer theory. Chapter 6 is devoted to the Riemann-Hilbert correspondence of renormalization, as dealt with in [*A. Connes* and *M. Marcolli*, Int. Math. Res. Not. 2004, No. 76, 4073–4091 (2004; Zbl 1131.81021)]. Chapter 7 describes two different approaches to a geometric description of dimensional regularization, the first being based on motivic notions while the second being based on techniques of noncommutative geometry. Chapter 8 is based on the author's [`arXiv:0804.4824`]. The final Chapter is concerned with two extensions of the theme of Feynman integrals and motives based on the parametric representation of Feynman integrals and their associated algebraic varieties, the first extension dealing with fermionic fields based on [*M. Marcolli* and *A. Rej*, J. Phys. A, Math. Theor. 41, No. 31, Article ID 315402, 21 p. (2008; Zbl 1156.81036)] while the second one being concerned with scalar quantum field theories over a noncommutative spacetime.

Last but not least, I must complain of the author's manner of citation. The author cites works in the reference as [Connes and Marcolli (2006b)]. This citation is very standard, but the reader cannot find such labels in the bibliography. This is highly perplexing, especially when the paper cited is still a preprint existing only in arXiv.

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

14-02	Research monographs (algebraic geometry)
81-02	Research monographs (quantum theory)
14C15	(Equivariant) Chow groups and rings; motives
81Q30	Feynman integrals and graphs; applications of algebraic topology and algebraic geometry
14A22	Noncommutative algebraic geometry
14J81	Relationships of algebraic surfaces with physics

Keywords:

Feynman integral; Feynman diagram; Tate motive; perturbative quantum field theory; period; oscillatory integral; Gelfand-Leray form; Connes-Kreimer theory; Radon transform; Hodge structure; noncommutative geometry; Galois group; supermanifold; Kirchhoff-Symaznik polynomial; dimensional regularization; BPHZ renormalization; Tannakian category; Grothendieck ring; monodromy; weight fibration; vanishing cycles; topological simplex; singularities; mixed Tate; tubular neighborhood; Kummer motive; Milnor fiber; motivic sheaves; normal crossings; Picard-Fuchs equation; Riemann-Hilbert correspondence; Hopf algebra; Igusa L-function

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