

**Iglesias-Zemmour, Patrick**

**Diffeology.** (English) [Zbl 1269.53003]

*Mathematical Surveys and Monographs* 185. Providence, RI: American Mathematical Society (AMS) (ISBN 978-0-8218-9131-5/hbk). xxiii, 439 p. (2013).

The category of smooth manifolds and smooth maps is by no means an adequate category for differential geometry. One of the crucial defects is that it fails to be Cartesian closed. Several enlarged categories are proposed in [Zbl 0501.58010; Zbl 0427.58005; Zbl 0498.58004; Zbl 0146.19402; Zbl 0389.58001; Zbl 0301.58006; Zbl 0227.58003]. These categories are thoroughly investigated with due regard to the relationship of one with another by [Zbl 1220.18013]. The category of Chen spaces and that of Souriau's diffeological spaces as well as the category of simplicial complexes are shown to be concrete sheaves on a concrete site in [Zbl 1237.58006], so that they enjoy many nice properties such as locally cartesian closedness, which the category of Frölicher spaces fails to enjoy.

The present book aims to propagate differential geometry on Souriau-Chen lines. It covers a wide range of differential geometry such as diffeological vector spaces, homotopy of diffeological spaces, differential forms and de Rham cohomology, diffeological groups, diffeological fiber bundles, and even symplectic diffeology. The book is undoubtedly to be welcomed by the general mathematical community. The author was once a student of Jean-Marie Souriau.

The book [The convenient setting of global analysis. Providence, RI: American Mathematical Society (AMS) (1997; Zbl 0889.58001)] by *A. Kriegl* and *P. W. Michor* is written under the strong influence of Frölicher spaces, though the authors are too contained to develop differential geometry of Frölicher spaces in full detail, succumbing easily to the orthodox philosophy of smooth manifolds. They have discussed convenient vector spaces (i.e., vector spaces within the category of Frölicher spaces enjoying some sort of completeness) only as building blocks for constructing infinite-dimensional manifolds. *A. Kock* and *G. E. Reyes* [Zbl 0596.18005; Zbl 0601.18006; Zbl 0634.18007] have investigated the relationship between the category of convenient vector spaces and a well-adapted model of synthetic differential geometry. Differential geometry of Frölicher spaces in the sense of the present book under review is being developed in [Zbl 1186.51007; Zbl 1193.51017; Zbl 1232.58001; Zbl 1242.58001; Zbl 1263.58001].

Homotopy theory is the study of the category of topological spaces and equivalence classes of continuous maps with respect to the relation of homotopy. As is well known, it is axiomatized by the notion of a model category. If this axiomatization is applied to the category of chain complexes over some commutative ring  $R$ , we get the derived category of  $R$ -modules. Differential geometry should be axiomatized in the same spirit, and it is being developed by *H. Nishimura* [Math. Appl., Brno 1, No. 2, 183–195 (2012; Zbl 1285.51010); *ibid.* 1, No. 2, 171–182 (2012; Zbl 1285.51009)].

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**MSC:**

- 53-02 Research monographs (differential geometry)
- 53C99 Global differential geometry
- 58A99 General theory of differentiable manifolds

Cited in **3** Reviews  
Cited in **33** Documents

**Keywords:**

diffeology, differential geometry; diffeological space; Chen space; smooth manifold; differential form; diffeological fiber bundle; diffeological vector space; diffeological group; symplectic diffeology