Cacciatori, Sergio L.; **Noja, Simone**; **Re, Riccardo**
Non-projected Calabi-Yau supermanifolds over $\mathbb{P}^2$. (English) Zbl 07124448
Math. Res. Lett. 26, No. 4, 1027-1058 (2019).

This paper is concerned with the obstruction of supermanifolds to be projected, concentrating on supermanifolds of a complex projective space $\mathbb{P}^n_{\mathbb{C}}$ as the associated reduced manifold and seeking to find the most general conditions for such supermanifolds to be non-projected. It is shown that only varieties of bosonic dimension 1 and 2 admit non-projected structures. The authors succeed in classifying non-projected supermanifolds over $\mathbb{P}^1$ of odd dimension 2. They provide a necessary and sufficient condition for the supermanifold over $\mathbb{P}^2$ to be non-projected. What is indeed remarkable, these supermanifolds turn out to be no other but Calabi-Yau supermanifolds [*S. Noja* et al., J. High Energy Phys. 2017, No. 4, Paper No. 94, 43 p. (2017; Zbl 1378.83088)]. It is established that these supermanifolds are not embeddable into a split projective superspace. The authors hold that the role of embedding spaces in supergeometry has to be played by super Grassmannians, establishing that all of the non-projected supermanifolds over $\mathbb{P}^2$ are embeddable into a certain super Grassmannian (Theorem 6.1).

The authors then investigate the supermanifolds corresponding to two distinct choices of the fermionic sheaf $\mathcal{F}_{\mathcal{M}}$, namely,

- the case when $\mathcal{F}_{\mathcal{M}}$ is the decomposable sheaf

$$\mathcal{O}_{\mathbb{P}^2}(-1) \oplus \mathcal{O}_{\mathbb{P}^2}(-2)$$

- the non-decomposable case when is the cotangent bundle $\Omega^1_{\mathbb{P}^2}$ of $\mathbb{P}^2$.

In the decomposable case, it is shown that it is neither projective nor Π-pprojective. In the non-decomposable case, it is demonstrated that it coincides with the Π-projective plane $\mathbb{P}^2_{\Pi}$ introduced by *Yu. I. Manin* [Gauge fields and complex geometry. (Kalibrovochnye polya i kompleksnaya geometriya) (Russian). Moskva: "Nauka". Glavnaya Redaktsiya Fiziko-Matematicheskoj Literatury. (1984; Zbl 0576.53002); Prog. Phys. 5, 231–234 (1982; Zbl 0535.70018)] in a completely different way. It is established that this unexpected correspondence is only a particular case of a general fact [*S. Noja*, J. Geom. Phys. 124, 286–299 (2018; Zbl 1388.58004)], meaning that Π-projective geometry arises naturally as one considers the cotangent bundle of projective spaces as the fermionic bundle over $\mathbb{P}^n$.

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MSC:

53D37 Mirror symmetry, symplectic aspects; homological mirror symmetry; Fukaya category
32Q25 Calabi-Yau theory
58C50 Analysis on supermanifolds or graded manifolds
58A50 Supermanifolds, etc. (global analysis)

Full Text: DOI