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The genuine operadic nerve. (English) Zbl 07104234

Theory Appl. Categ. 34, 736-780 (2019).

For any simplicial operad $\mathcal{O} \in \mathrm{sOp}$, *J. P. May* and *R. Thomason* [Topology 17, 205–224 (1978; Zbl 0391.55007)] constructed an associated simplicial category $\mathcal{O}^{\otimes}$, living over the category $\boldsymbol{F}_*$ of pointed finite sets, called the *category of operators*, showing that the theory of algebras over $\mathcal{O}$ and $\mathcal{O}^{\otimes}$ coincide.

The homotopy coherent nerve of is denoted $N^{\otimes}(\mathcal{O})$, called the *operadic nerve* by *J. Lurie* ["Higher algebra", Preprint, `http://www.math.harvard.edu/~lurie/papers/HA.pdf`, 2.1.1.27]. The principal objective in this paper is to generalize the story of the operadic nerve to the equivariant setting, incorporating the action of a finite group G. The source of the new map is the category sOp_G of *simplicial genuine equivariant operads* introduced in [the author and *L. A. Pereira*, "Genuine equivariant operads", Preprint, `arXiv:1707.02226`] as a generalization of simplicial G-operad, while it finds its target in the fantastic theory of parametrized ∞-categories and parametrized homotopy theory of [*C. Barwick* et al., "Parametrized higher category theory and higher algebra: A general introduction", Preprint, `arXiv:1608.03654`].

The principal results of this paper are extensions of [Lurie, loc. cit., Propositions 2.1.1.27 and 4.1.7.10], providing a 1-categorical translation between these two theories of homotopical equivariant operads.

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

55P91 Equivariant homotopy theory
55P48 Loop space machines, operads
19D23 Symmetric monoidal categories (K-theory)
18D30 Fibered categories

Keywords:

infinity operads; equivariant operads; symmetric monoidal categories

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