Reutter, David Jakob; Vicary, Jamie

Biunitary constructions in quantum information. (English) Zbl 1419.18008

High. Struct. 3, 109-154 (2019).

Biunitary connections or *biunitaries* were introduced by *A. Ocneanu* [Lond. Math. Soc. Lect. Note Ser. 136, 119–172 (1988; Zbl 0696.46048)] as a key tool in the study on classification of subfactors. The principal objective in this paper is to apply these gadgets to construct a unitary error basis that cannot be built by using any previously known method. The authors use an approach to biunitaries developed by *D. Bisch* and *V. Jones* [Duke Math. J. 101, No. 1, 41–75 (2000; Zbl 1075.46053); Adv. Math. 175, No. 2, 297–318 (2003; Zbl 1041.46048)], *V. F. R. Jones* [Ser. Knots Everything 24, 94–117 (2000; Zbl 1021.46047)], *V. F. R. Jones* et al. [Bull. Am. Math. Soc., New Ser. 51, No. 2, 277–327 (2014; Zbl 1301.46039)], *S. Morrison* and *E. Peters* [Int. J. Math. 25, No. 8, Article ID 1450080, 51 p. (2014; Zbl 1314.46074)] within the theory of planar algebras investigating the linear representation theory of algebraic structures in the plane. The paper consists of 5 sections. The type of a biunitary is the shading pattern surrounding the vertex. The authors show in §2 that a variety of structures in quantum information theory correspond exactly to biunitaries of particular type. The main results in the paper count on the simple fact that the diagonal composite of two biunitaries is again biunitary. The authors show in §3 that, given the description of quantum combinatorial structures in terms of biunitaries, one can at once write down a plethora of schemes for the construction of certain quantum structures from others. §4 is devoted to the problem of equivalence, giving a new generic definition of equivalence for biunitaries and showing that it recovers precisely usual notions of equivalence for each of the quantum structures considered here. §5 is dedicated to construction of a unitary error basis $\mathcal{U}$ lying within the biunitary composition method being by no means quantum shift-and-multiply or algebraic.

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

18D05 Double categories, 2-categories, bicategories and generalizations
15B34 Boolean and Hadamard matrices
81P45 Quantum information, communication, networks

Keywords:

Hadamard matrices; unitary error bases; quantum Latin squares; biunitary; 2-category; graphical calculus; planar algebra

Full Text: Link

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