Baez, John C.; **Fong, Brendan**
A compositional framework for passive linear networks. (English) ⸢Zbl 1402.18005⸥
Theory Appl. Categ. 33, 1158-1222 (2018).

This paper is concerned, from a compositional viewpoint, with electric circuits from passive linear components in the sense that

(1) components do not produce energy (i.e., excluding batteries and current sources),

(2) components respond linearly to an applied voltage (i.e., excluding nonliear resistors or diodes), and

(3) components are provided with one input and one output (so that circuits are describable by graphs with edges labelled by components).

The most typical components considered in this paper are linear resistors, capacitors and inductors.

The authors construct the category Circ whose objects are finite sets and whose morphisms are open circuits with several inputs and several outputs, the composition being the resulting union of labelled graphs by identification of the outputs of one circuit and the inputs of the next. Each such circuit gives rise to a Dirichlet form, which is shown to determine the externally observable behavior of the circuit. The authors also construct the category LagRel whose objects are finite sets and whose morphisms are Lagrangian relations (That is to say, a morphism $f : X \to Y$ is a Lagrangian subspace of $\overline{\mathbb{F}^X \oplus (\mathbb{F}^X)^* \oplus \mathbb{F}^Y \oplus (\mathbb{F}^Y)^*}$). It is proved that both categories are symmetric monoidal categories with some extra structure known as hypergraph categories (§4.2).

The main result in this paper (§7) is the existence of a hypergraph functor called the black box functor

$$\blacksquare : \mathrm{Circ} \to \mathrm{LagRel}$$

More general circuits are treated in a companion paper [*J. C. Baez* et al., Theory Appl. Categ. 33, 727–783 (2018; Zbl 1400.18004)].

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

18C10	Theories, structure, and semantics
18D10	Monoidal, symmetric monoidal and braided categories
53D12	Lagrangian submanifolds; Maslov index
94C05	Analytic circuit theory

Cited in **1** Review
Cited in **12** Documents

Keywords:

passive linear network; electric circuit; principle of minimum power; black box; decorated cospan; compact closed category; hypergraph category; Lagrangian relation

Software:

DYNAMO

Full Text: Link

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