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Classical field theory. On electrodynamics, non-Abelian gauge theories and gravitation. (English) Zbl 1246.83001

Graduate Texts in Physics. Berlin: Springer (ISBN 978-3-642-27984-3/hbk; 978-3-642-27985-0/ebook). xii, 433 p. (2012).

This book consists of six chapters. The first four chapters deal with Maxwell theory. Starting from Maxwell's equations in integral form, that is to say, from the phenomenological and experimentally verified basis of electrodynamics, the local equations are formulated and discussed with their general time and space dependence right from the start. Static or stationary situations appear as special cases for which Maxwell's equations split into two more or less independent groups and thus are decoupled to a certain extent. Great importance is attached to the symmetries of the Maxwell's equations, in particular, their covariance with respect to Lorentz transformations. The venerable vector and tensor analysis designed for three-dimensional Euclidean spaces is generalized to higher dimensions and to Minkowski signature. While the exterior product is the generalization of the vector product in $\mathbb{R}^3$, Cartan's exterior derivative is the natural generalization of the familiar operations of divergence, curl and gradient. Another central characteristic of the book is the treatment of electrodynamics in the framework of classical field theory by means of a Lagrange density and Hamilton's principle. As in mechanical systems, the theorem of Noether yields the relationship between invariance of the Lagrangian density under transformations in space and time and conservation laws. Among the many applications of Maxwell theory, the author chose some characteristic and nowadays particularly relevant examples such as an extensive discussion of polarization of electromagnetic waves, the description of Gaussian beams, and optics of metamaterials with negative index of refraction.

The fifth chapter studies the generalization of the concept of a locally invariant gauge theory to non-Abelian gauge groups constructed by following the model of Maxwell theory. The Yang-Mills theories as they are called today are essential and indispensible for our present understanding of the fundamental interactions of nature.

The last chapter develops the geometric foundations for Einstein's equations for gravitation. These equations relate the geometry of spacetime with the energy-momentum content of matter and radiation present in the universe. One studies characteristic solutions of Einstein's equations and analyzes their properties.

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

83-01	Textbooks (relativity)
78A25	General electromagnetic theory
00A79	Physics
81T13	Yang-Mills and other gauge theories
83C40	Gravitational energy and conservation laws; groups of motions
83C05	Einstein's equations (general structure, canonical formalism, Cauchy problems)
83C22	Einstein-Maxwell equations
83A05	Special relativity
81V22	Unified theories of elementary particles
70H40	Relativistic dynamics
83C15	Closed form solutions of equations in general relativity

Keywords:

theorem of Noether; classical field theory; Yang-Mills theory; general relativity; Einstein's equations; Maxwell's equations; Lagrangian density; Hamilton's principle; electrodynamics

Full Text: DOI