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Mathematical gauge theory. With applications to the standard model of particle physics.
(English) Zbl 1390.81005
Universitext. Cham: Springer (ISBN 978-3-319-68438-3/pbk; 978-3-319-68439-0/ebook). xviii, 657 p. (2017).

Assuming an introductory course on differential geometry and some basic knowledge of special relativity, both of which are summarized in the appendices, the book expounds the mathematical background behind the well-established standard model of modern particle and high energy physics. The book is based upon the author's lecture notes for graduate courses at the university of Stuttgart and the LMU Munich in Germany. The first six chapters of the book discuss mathematical foundations, while the remaining three chapters are concerned with the standard model of elementary particle physics. I believe that the book will be a standard textbook on the standard model for mathematics-oriented students.

The standard model of elementary particles is a gauge theory with Lie group

$$\mathbf{SU}(3) \times \mathbf{SU}(2) \times \mathbf{U}(1)$$

while grand unified theories are based on such Lie groups as

$$\mathbf{SU}(5).$$

The first chapter gives basic concepts about Lie groups and Lie algebras, including correspondences between Lie groups and Lie algebras as well as Cartan's theorem on closed subgroups on the lines of [*H. Baum*, Eichfeldtheorie. Eine Einführung in die Differentialgeometrie auf Faserbündeln. 2nd revised ed. Heidelberg: Springer Spektrum (2014; Zbl 1297.53001); Eichfeldtheorie. Eine Einführung in die Differentialgeometrie auf Faserbündeln. Berlin: Springer (2009; Zbl 1175.53001); *T. Bröcker* and *T. tom Dieck*, Representations of compact Lie groups. Corrected reprint of the 1985 orig. New York, NY: Springer (1995; Zbl 0874.22001); Representations of compact Lie groups. New York: Springer (1985; Zbl 0581.22009)].

Three Dirac spinors for each quark flavor gather together to form a vector in a representation space $\mathbb{C}^3$ of the gauge group $\mathbf{SU}(3)$ of quantum chromodynamics, while two left-handed Weyl spinors gather together to form a vector in a representation space $\mathbb{C}^2$ of the gauge group $\mathbf{SU}(2)\times\mathbf{U}(1)$ of the electroweak interaction. The second chapter describes representations of Lie groups and Lie algebras in general as well as the structure of semisimple and compact Lie algebras. The third chapter discusses Lie group action on manifolds.

It is a principal bundle over spacetime with structure group given by the gauge group that is the fundamental picture in a gauge theory. With the back ground knowledge of Lie groups, Lie algebras, representations and group actions, Chapter 4 addresses fiber bundles in general, and more specifically, principal bundles, vector bundles and associated bundles, all of which constitute the stage of gauge theory. Chapter 5 discusses connections on principal bundles corresponding to gauge fields, whose particle excitations in the associated quantum field theory are gauge bosons transmitting interactions, while matter fields in the standard model such as quarks and leptons correspond to sections of vector bundles associated to the principal bundle as well as fermions are twisted by spinor bundles.

Chapter 6 discusses spinors, which do not transform directly under the orthogonal group, but under a certain double covering called the (orthochronous) spin group. The discussion of spinors in many mathematical expositions are restricted to the Riemannian case (notable exception being [*H. Baum*, Spin-Strukturen und Dirac-Operatoren über pseudoriemannschen Mannigfaltigkeiten. Leipzig: BSB B. G. Teubner Verlagsgesellschaft (1981; Zbl 0519.53054)]), but this chapter addresses orthogonal groups, Clifford algebras, spin groups and spinors in the Lorentzian and general pseudo-Riemannian cases, because the author has applications to physics in mind.

Lagrangians are to be considered the fundamental cornerstones of field theories. Lagrangians which are harmonic correspond to free theories, while Lagrangians which involve anharmonic terms of order three or higher in the fields give rise to the quantum field theory with creation and annihilation of particles and thus to interactions. Feynman diagrams depict interactions between fields. The Lagrangians of physical importance are mainly governed by three principles, namely, (1) existence of symmetries, (2) the quantum field theory is to be renormalizable, and (3) the quantum field theory is to be free of gauge anomalies. Chapter 7 discuss briefly how these three principles restrict the possible variety of Lagrangians and then investigate the Lagrangians appearing in the standard model of elementary particles, namely, (1) the Yang-Mills Lagrangian, (2) the Klein-Gordon and Higgs Lagrangian, (3) the Dirac Lagrangian, and (4) Yukawa coupling. The exposition of Chapter 7 and Chapter 8 depends on [*D. Bleecker*, Gauge theory and variational principles. Reading, MA, etc.: Addison-Wesley Publishing Company, Inc. (1981; Zbl 0481.58002); *S. Weinberg*, The quantum theory of fields. Vol. 1: Foundations. Corr. Repr. of the 1995 orig. Cambridge: Cambridge University Press (1996; Zbl 0959.81002); The quantum theory of fields. Volume II: Modern applications. Cambridge: Cambridge University Press (1996; Zbl 0885.00020); The quantum theory of fields. Vol. 3: Supersymmetry. Cambridge: Cambridge University Press (2000; Zbl 0949.81001); The quantum theory of fields. Vol. 3. Supersymmetry. Cambridge: Cambridge University Press (2005; Zbl 1069.81501)].

The principal objective in Chapter 8 is to apply the formalism of mathematical gauge theory to physics. The author first exposes gauge theories in which the gauge symmetry is broken spontaneously, giving rise to one or several Higgs bosons. He also discusses (1) the particle content and the representation of the gauge group, (2) the Higgs mechanism of mass generation (considering the case of a general Lie group and Higgs vector space with possibly several Higgs bosons as well as the specific case of the standard model with a single Higgs boson), and (3) the explicit Lagrangian concerning the interaction among all known elementary particles.

Chapter 9, consisting of 7 sections, deals with some advanced topics in particle physics and modern developments beyond the standard model. §9.1 addresses flavor symmetry and chiral symmetry breaking in QCD, referring the reader to [*N. Brambilla* et al., "QCD and strongly coupled gauge theories: challenges and perspectives", Eur. Phys. J. C 74, No. 10, Article No. 2981 (2014; `doi:10.1140/epjc/s10052-014-2981-5`); *R. S. Chivukula*, "The origin o mass in QCD", Preprint, `arXiv:hep-ph/0411198`; *S. Dürr* et al., "Ab initio determination of light hadron masses", Science 322, No. 5905, 1224–1227 (2008; `doi:10.1126/science.1163233`); *Sz. Borsanyi* et al., "Ab initio calculation of the neutron-proton mass difference", Science 347, No. 1452, 1452–1455 (2015; `doi:10.1126/science.1257050`)] for further reading. §9.2 is concerned with the question how mass terms of neutrinos can be added to the standard model as well as the phenomenon of neutrino oscillations, particularly discussing one natural explanation called the seesaw mechanism after [*C. Giunti* and *C. W. Kim*, Fundamentals of Neutrino Physics and Astrophysics. Oxford: Oxford University Press (2007); *M. Thomson*, Modern Particle Physics. Cambridge: Cambridge University Press (2013)]. §9.3 only addresses CP violation coming from quark mixing and the weak interaction on the lines of [*G. C. Branco* et al., CP Violation. Oxford: Oxford University Press (1999)]. §9.4 discusses running coupling constants, and §9.5 studies the grand unified theories described by the simple Lie groups $\mathbf{SU}(5)$ and $\mathbf{Spin}(10)$, following the mathematical reference [Zbl 1196.81252] and the physical reference [*R. N. Mohapatra*, in: Particle physics. Proceedings of the summer school ICTP, Trieste, Italy, June 21–July 9, 1999. Singapore: World Scientific. 336–394 (2000; Zbl 0970.81076)]. §9.6 studies a minimal supersymmetric standard model, referring the reader to [Zbl 0984.00503; *R. N. Mohapatra*, Unification and Supersymmetry. New York/Berlin/Heidelberg: Springer (2003)]. §9.7 provides exercises for Chapter 9.

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MSC:

81-02	Research monographs (quantum theory)
81T13	Yang-Mills and other gauge theories
53C07	Special connections and metrics on vector bundles (Hermite-Einstein-Yang-Mills)
81T40	Two-dimensional field theories, conformal field theories, etc.
81V22	Unified theories of elementary particles
55Q91	Equivariant homotopy groups
81R25	Spinor and twistor methods in quantum theory
81T18	Feynman diagrams
70S05	Lagrangian formalism and Hamiltonian formalism
81R40	Symmetry breaking (quantum theory)

Full Text: DOI