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Symplectic geometry of geometric optics. (Japanese) Zbl 1386.53001

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In the 19th century physics culminated in Maxwell's electromagnetism (`http://rstl.royalsocietypublishing.org/content/155/459`), which turned out to be a theory of visible lights as electromagnetic radiation (consisting of electromagnetic waves, which are no other than synchronized oscillations of electric and magnetic fields). Maxwell's classical electromagnetism was to be superseded by quantum electromagnetism in the middle of the 20th century. Maxwellian electromagnetism is a good approximation to quantum electromagnetism, as far as we consent to do without quantum effects. *Wave optics*, also called *physical optics*, lies somewhere between classical electromagnetism and geometric optics. *Geometric optics* is a classical approximation to wave optics, describing light propagation in terms of rays. This is valid only in case when the dimensions of the various apertures are tremendously large in comparison with the wavelength of the light and besides we gladly agree to grudgingly refrain from examining scrupulously what is happening in the proximity of shadows or foci. Geometric optics presupposes five principles: (1) light keeps straight on in a homogeneous medium; (2) the law of reflection; (3) Snell's law; (4) if a ray of light travels from point A to point B, then it can travel reversely from B to A, (5) two rays of light are completely independent even when they intersect with each other. *Linear optics* is in turn an approximation to geometric optics, holding only in the case when the various angles under consideration are so small that we can enjoy approximate identities such as

$$\sin\theta \doteq \theta, \cos\theta \doteq 1, \tan\theta \doteq \theta$$

in which we assume that the index of refraction is constant between refracting surfaces, while in general geometric optics we assume oppositely that the index of refraction is smoothly varying. *Gaussian optics* is a significant exemplification of linear optics under the assumption that all surfaces under consideration are rotationally symmetric around a central axis, in which we are engaged in tracing the trajectory of the ray of light as it passes through the various refracting surfaces or is reflected by diverse reflecting surfaces.

The first chapter of the classical book [*V. Guillemin* and *S. Sternberg*, Symplectic techniques in physics. Cambridge etc.: Cambridge University Press. (1984; Zbl 0576.58012)] deals with geometric optics from a symplectic standpoint. Roughly speaking, the book under review, inspired by the first chapter of [loc. cit.], can be regarded as a detailed account of it. The book consists of 6 chapters with an appendix on preliminary symplectic techniques. The first chapter is concerned with Fermat's principle or the principle of least time. The second chapter is engaged in Hamiltonian optics, which is geometric optics in terms of Hamiltonian mechanics. The third chapter deals with the Hamilton-Jacobi equation, beginning with Malus and Dupin's theorem (claiming that a group of rays preserves its normal congruence after any number of reflections and refractions) and then turning to the eikonal equation (a non-linear partial differential equation linking physical optics and geometric optics) and finally concludes with Huygens' principle, from which Fermat's principle follows mathematically.

The fourth chapter addresses linear optics and the theory of imaging in terms of symplectic mappings. The fifth chapter discusses aberrations and caustics to go beyond Gaussian optics, where the five Seidel aberrations are discussed (cf. [*A. O. Allen*, Proceedings Leeds 1 (1929); 392–401, 475–483 (1929; JFM 57.0993.05)]). The sixth chapter addresses the similarity between geometric optics and particle physics. It is argued that geometric optics and particle dynamics are the limits of wave optics and wave dynamics respectively as $\lambda_0 \to 0$ or $\hbar \to 0$, where λ_0 is the wave length in vacuum. Just as quantization is the transmogrification of classical dynamics to quantum mechanics, the author considers the transmogrification of geometric optics to wave optics. This chapter concludes with path integrals, by which we can understand where the Fermat principle and the Hamilton principle come from.

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53D05 Symplectic manifolds, general
53Z05 Applications of differential geometry to physics
78A05 Geometric optics

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geometric optics; linear optics; Gaussian optics; wave optics; symplectic geometry; electromagnetism; Hamiltonian optics; Fermat's principle; Hamilton-Jacobi equation; Malus and Dupin's theorem; Huygens' principle; Seidel abberrations; path integral; quantization