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Mathematical methods in Feynman path integrals. (Feynman keiro sekibun no sugakuteki houhou.) (Japanese) Zbl 1370.81002

Tokyo: Maruzen (ISBN 978-4-621-06451-1). 277 p. (2012).

The principal purpose of this book is to show that the Feynman path integral converges, as long as the passage of time is short enough and the potential abides by certain conditions. The book is divided into two parts.

Part I consists of 7 chapters. The first chapter explains Feynman's original idea of path integral without taking care of mathematical rigor. Chapter 2 shows the existence of classical orbits by solving the variational problem. Chapter 3 gives preliminary acquaintances with methods of oscillatory integral operators or Fourier integral operators for comprehension of Feynman path integrals. Chapter 4 shows that the oscillatory integral converges so long as the potential satisfies certain conditions and the passage of time is short enough, so that the stationary phase method can be applied. Chapter 5 is concerned with the proof that the Feynman path integral dealt with in the previous chapter indeed converges. Chapter 6 establishes the theorem that the integral transform $K(v,t,s)$ derived from the Feynman path integral is really the fundamental solution of the Schrödinger equation. In this process it is shown, completely different from [*G. D. Birkhoff*, Bull. Am. Math. Soc. 39, 681–700 (1933; JFM 59.1530.01); Bull. Am. Math. Soc. 39, 681–700 (1933; Zbl 0008.08902)] and [*V. P. Maslov*, Théorie des perturbations et méthodes asymptotiques. Suivi de deux notes complementaires de V. I. Arnol'd et V. C. Bouslaev. Traduit par J. Lascoux et R. Seneor. Paris: Dunod; Paris: Gauthier-Villars (1972; Zbl 0247.47010)], that the principal term obtained by applying the stationary phase method to the Feynman path integral satisfies the equation of continuity. Chapter 7 argues that, even if the passage of time is long, the difficulty in integrals in infinite-dimensional spaces finally disappears, but the difficulty in calculus of variations in the large is looming instead.

Part II, consisting of 7 chapters (Chapter 8-Chapter 14), is supplements from real analysis. Chapter 10, based on the author's [Nagoya Math. J. 124, 61–98 (1991; Zbl 0728.41031)], establishes the stationary phase method on a space of large dimension, which has played a cardinal role in the discussions of Chapter 5. The crucial point in the proof lies in *H. Kumano-go* and *K. Taniguchi*'s theorem [Funkc. Ekvacioj, Ser. Int. 22, 161–196 (1979; Zbl 0568.35092)], which is dealt with by a different method [*D. Fujiwara* et al., Funkc. Ekvacioj, Ser. Int. 40, No. 3, 459–470 (1997; Zbl 0897.35086)] in Chapter 8. Chapter 9 revisits the stationary phase method. Chapter 11 is a simplified presentation of [*K. Asada* and *D. Fujiwara*, Jpn. J. Math., New Ser. 4, 299–361 (1978; Zbl 0402.44008)]. Chapter 12 presents the definition of distributions and their fundamental properties. Chapter 13 deals with Hadamard's global inverse function theorem [*J. Hadamard*, Bull. Soc. Math. Fr. 34, 71–84 (1906; JFM 37.0672.02)] after [*J. T. Schwartz*, Nonlinear functional analysis. Notes by H. Fattorini, R. Nirenberg and H. Porta. With an additional chapter by Hermann Karcher. (Notes in Mathematics and its Applications.) New York-London-Paris: Gordon and Breach Science Publishers. (1969; Zbl 0203.14501)]. Chapter 14 is an epilogue.

A minor complaint of the reviewer is that the author should be more careful in writing a book. Kumanogo should have been written Kumano-go, though the author has written papers with Kumano-go. The author should have written the title of his own paper correctly. He has written a paper entitled "A proof of estimates of …" in place of "A proof of estimate of …". He quoted Maslov's book wrongly by number [19] in place of number [20].

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

81-02	Research monographs (quantum theory)
81Q30	Feynman integrals and graphs; applications of algebraic topology and algebraic geometry
81S40	Path integrals in quantum mechanics
58D30	Spaces and manifolds of mappings in applications to physics
00A79	Physics