Baez, John C.; Coya, Brandon; Rebro, Franciscus

Props in network theory. (English) Zbl 1400.18004

Theory Appl. Categ. 33, 727-783 (2018).

It was *F. W. Lawvere* [Proc. Natl. Acad. Sci. USA 50, 869–872 (1963; Zbl 0119.25901); Repr. Theory Appl. Categ. 2004, No. 5, 1–121 (2004; Zbl 1062.18004)] that introduced functorial semantics in 1963. *S. MacLane* [Bull. Am. Math. Soc. 71, 40–106 (1965; Zbl 0161.01601)] introduced props or PROPs as an extension of Lawvere's functorial semantics, the acronym standing for "PROducts and Permutations". Feynman diagrams [*R. P. Feynman*, Phys. Rev., II. Ser. 76, 749–759 (1949; Zbl 0037.12406)], as far as just one type of particle is concerned, are no other than props where an operation describes a process with several particles coming in and several particles going out. It was [*A. Joyal* and *R. Street*, Adv. Math. 88, No. 1, 55–112 (1991; Zbl 0738.18005)] that enabled one to understand that Feynman diagrams are in a nutshell a method of depicting morphisms in symmetric monoidal categories. Fortunately props are now in the arsenal of almost every mathematical physicist.

However, props are virtually unknown in engineering, though the idea of Feynman diagrams was preceded and developed in engineering [*H. F. Olson*, Dynamical analogies. New York, NY: Van Nostrand (1943); *H. M. Paynter*, Analysis and design of engineering systems. Cambridge, MA: MIT Press (1961)], which propagated in economics and biology [*J. W. Forrester*, Industrial dynamics. Cambridge, MA: MIT Press (1961); *H. T. Odum*, Ecological and general systems. New York, NY: Wiley (1984)]. The principal objective in this paper is to illustrate the usefulness of props in electrical circuits as an example of various possible applications of props [*D. C. Kanopp* et al., System dynamics. New York, NY: Wiley (1990); *F. T. Brown*, Engineering system dynamics. New York, NY: Taylor and Francis (2007)].

The authors give a shorter new construction of the black-boxing functor in ["A compositional framework for passive linear networks", Preprint, `arXiv:1504.05625`, Theorem 1.1]. Any circuit made of linear resistors, inductors and capacitors obey a sort of generalization of the principle of minimum power, and any system governed by a minimum principle makes the black-boxing a functor to some category where the morphisms are Lagrangian relations [*J. C. Baez* et al., J. Math. Phys. 57, No. 3, 033301, 30 p. (2016; Zbl 1336.60147), §13].

The paper consists of 11 sections. Starting (§3) with circuits made only of ideal perfectly conductive wires, which are regarded as morphisms in a prop called Circ, the authors construct a black-boxing functor

$$\blacksquare : \mathrm{Circ} \to \mathrm{LagRel}_k$$

in §8, where LagRel_k with k an arbitary field is a prop with symplectic vector spaces of the form k^{2n} as objects and linear Lagrangian relations as morphisms, and $\blacksquare$ is a morphism of props. In §9 the authors extend black-boxing to a larger class of circuits including linear registors, inductors and capacitors. In §10 it is explained how electric circuits are related to signal-flow diagrams used in control theory. The final section (§11) is devoted to investigation of nonlinear circuits.

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

18C10	Theories, structure, and semantics
18D10	Monoidal, symmetric monoidal and braided categories
94C05	Analytic circuit theory
94C15	Applications of graph theory to circuits and networks

Keywords:

circuit; functorial semantics; network; PROP; symmetric monoidal category

Software:

DYNAMO

Full Text: Link

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