Baez, John C.; **Huerta, John**
An invitation to higher gauge theory. (English) ‖ Zbl 1225.83001 ‖
Gen. Relativ. Gravitation 43, No. 9, 2335-2392 (2011).

Higher gauge theory is a generalization of the familiar gauge theory, which is concerned with transports of point particles, to higher-dimensional objects. It should not be surprising that the emerging theory should be applicable to string theory and loop quantum gravity, both of which agree that we need higher-dimensional extended objects, though always disputing in almost all other points concerned. This paper is to sketch how to generalize the theory of parallel transport from point particles to 1-dimensional objects with such a bare minimum of such prerequisites as manifolds, differential forms, Lie groups, Lie algebras and the traditional theory of bundles and connections. In place of a connection, which tells us how particles transform as they move along paths, one should speak of a 2-connection, which tells us how strings transform as they sweep out surfaces. Six interesting examples, such as the Poincaré 2-group leading to spin foam model for Minkowski spacetime, are discussed. For more applications, one can visit, e.g., [*H. Sati*, Proceedings of Symposia in Pure Mathematics 81, 181–236 (2010; Zbl 1210.81089), J. Aust. Math. Soc. 90, No. 1, 93–108 (2011; Zbl 1217.81131)].

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MSC:

83-02	Research monographs (relativity)
81-02	Research monographs (quantum theory)
83A05	Special relativity
83D05	Relativistic gravitational theories other than Einstein's
83C22	Einstein-Maxwell equations
83E30	String and superstring theories
78A25	General electromagnetic theory
17B45	Lie algebras of linear algebraic groups
83C05	Einstein's equations (general structure, canonical formalism, Cauchy problems)
81T20	Quantum field theory on curved space backgrounds
83E50	Supergravity

Cited in **1** Review
Cited in **33** Documents

Keywords:

category; gerbe; higher gauge theory; string; 2-category; 2-group

Full Text: DOI   arXiv

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