

**Gálvez-Carrillo, Imma; Kock, Joachim; Tonks, Andrew**

**Decomposition spaces, incidence algebras and Möbius inversion. III: The decomposition space of Möbius intervals.** (English) [Zbl 1403.18017](#)

*Adv. Math.* 334, 544-584 (2018).

The paper under review is the third part of a trilogy devoted to the investigation of decomposition spaces and their incidence algebras. The first part [ibid. 331, 952–1015 (2018; [Zbl 1403.00023](#))] introduced the notion of decomposition space as a general framework for incidence algebras and Möbius inversion, discovered independently in a different guise by *T. Dyckerhoff* and *M. Kapranov* [“Higher Segal spaces I”, [arXiv:1212.3563](#)] who were motivated by geometry, representation theory and homological algebra. The second part [ibid. 333, 1242–1292 (2018; [Zbl 1403.18016](#))], by imposing suitable finiteness conditions on decomposition spaces, arrived at Möbius decomposition spaces as a far-reaching generalization of the notion of Möbius category in [*P. Leroux*, *Cah. Topologie Géom. Différ. Catégoriques* 16, 280–282 (1976; [Zbl 0364.18001](#))]. This third part, subsuming discoveries of *F. W. Lawvere* and *M. Menni* [*Theory Appl. Categ.* 24, 221–265 (2010; [Zbl 1236.18001](#))], introduces the Möbius decomposition space of Möbius intervals, which is shown to be a universal Möbius decomposition space in a precise sense, though its proof is highly fastidious.

Reviewer: [Hirokazu Nishimura \(Tsukuba\)](#)

**MSC:**

[18G30](#) Simplicial sets; simplicial objects in a category  
[16T10](#) Bialgebras  
[06A11](#) Algebraic aspects of posets  
[16T05](#) Hopf algebras and their applications  
[55U35](#) Abstract homotopy theory; axiomatic homotopy theory

Cited in **1** Review  
Cited in **2** Documents

**Keywords:**

decomposition space; 2-Segal space; CULF functor; Möbius interval; Möbius inversion

**Full Text:** [DOI](#)

**References:**

- [1] Berger, C.; Melliès, P.-A.; Weber, M., Monads with arities and their associated theories, *J. Pure Appl. Algebra*, 216, 2029-2048, (2012) · [Zbl 1256.18004](#)
- [2] Carboni, A.; Lack, S.; Walters, R. F.C., Introduction to extensive and distributive categories, *J. Pure Appl. Algebra*, 84, 145-158, (1993) · [Zbl 0784.18001](#)
- [3] Cartier, P.; Foata, D., Problèmes combinatoires de commutation et réarrangements, *Lecture Notes in Mathematics*, vol. 85, (1969), Springer-Verlag Berlin, New York, republished in the “books” section of the Séminaire Lotharingien de Combinatoire · [Zbl 0186.30101](#)
- [4] Content, M.; Lemay, F.; Leroux, P., Catégories de Möbius et functorialités: un cadre général pour l’inversion de Möbius, *J. Combin. Theory Ser. A*, 28, 169-190, (1980) · [Zbl 0449.05004](#)
- [5] Dyckerhoff, T.; Kapranov, M., Higher Segal spaces I, *Lecture Notes in Math.*, to appear
- [6] Gálvez-Carrillo, I.; Kock, J.; Tonks, A., Decomposition spaces, incidence algebras and Möbius inversion, (old omnibus version, not intended for publication), preprint · [Zbl 1403.00023](#)
- [7] Gálvez-Carrillo, I.; Kock, J.; Tonks, A., Homotopy linear algebra, *Proc. Roy. Soc. Edinburgh A*, 148, 293-325, (2018) · [Zbl 06854604](#)
- [8] Gálvez-Carrillo, I.; Kock, J.; Tonks, A., Decomposition spaces, incidence algebras and Möbius inversion I: basic theory, *Adv. Math.*, 331, 952-1015, (2018) · [Zbl 1403.00023](#)
- [9] Gálvez-Carrillo, I.; Kock, J.; Tonks, A., Decomposition spaces, incidence algebras and Möbius inversion II: completeness, length filtration, and finiteness, *Adv. Math.*, 333, 1242-1292, (2018) · [Zbl 1403.18016](#)
- [10] Gálvez-Carrillo, I.; Kock, J.; Tonks, A., Decomposition spaces and restriction species, preprint

- [11] Gálvez-Carrillo, I.; Kock, J.; Tonks, A., Decomposition spaces in combinatorics, preprint
- [12] Gepner, D.; Haugseng, R.; Kock, J.,  $\infty$ -operads as analytic monads, preprint
- [13] Glasman, S., A spectrum-level Hodge filtration on topological Hochschild homology, *Selecta Math.*, 22, 1583-1612, (2016) · [Zbl 1371.18011](#)
- [14] Illusie, L., *Complexe cotangent et déformations. II*, *Lecture Notes in Mathematics*, vol. 283, (1972), Springer-Verlag Berlin · [Zbl 0238.13017](#)
- [15] Joni, S. A.; Rota, G.-C., Coalgebras and bialgebras in combinatorics, *Stud. Appl. Math.*, 61, 93-139, (1979) · [Zbl 0471.05020](#)
- [16] Joyal, A., Foncteurs analytiques et espèces de structures, (*Combinatoire énumérative, Montréal/Québec, 1985*, *Lecture Notes in Mathematics*, vol. 1234, (1986), Springer Berlin), 126-159
- [17] A. Joyal, Disks, duality and  $\Theta$ -categories, September 1997.
- [18] Joyal, A., Quasi-categories and kan complexes, *J. Pure Appl. Algebra*, 175, 207-222, (2002) · [Zbl 1015.18008](#)
- [19] Joyal, A., The theory of quasi-categories, (*Advanced Course on Simplicial Methods in Higher Categories*, vol. II, *Quaderns*, vol. 45, (2008), CRM Barcelona), available at
- [20] Lawvere, F. W., Ordinal sums and equational doctrines, (Eckmann, B., *Seminar on Triples and Categorical Homology Theory*, ETH 1966/67, *Lecture Notes in Mathematics*, vol. 80, (1967), Springer-Verlag New York), *Theory Appl. Categ.*, 18, 1-303, (2008), available from · [Zbl 0165.03204](#)
- [21] F.W. Lawvere, State categories and response functors. Dedicated to Walter Noll, preprint, May 1986.
- [22] Lawvere, F. W.; Menni, M., The Hopf algebra of Möbius intervals, *Theory Appl. Categ.*, 24, 221-265, (2010) · [Zbl 1236.18001](#)
- [23] Leroux, P., LES catégories de Möbius, *Cah. Topol. Géom. Différ.*, 16, 280-282, (1976) · [Zbl 0364.18001](#)
- [24] Lurie, J., Higher topos theory, *Annals of Mathematics Studies*, vol. 170, (2009), Princeton University Press Princeton, NJ, available from · [Zbl 1175.18001](#)
- [25] Lurie, J., Higher algebra, (2013), available from
- [26] Rota, G.-C., On the foundations of combinatorial theory. I. theory of Möbius functions, *Z. Wahrsch. Verw. Gebiete*, 2, 340-368, (1964) · [Zbl 0121.02406](#)
- [27] Segal, G., Configuration-spaces and iterated loop-spaces, *Invent. Math.*, 21, 213-221, (1973) · [Zbl 0267.55020](#)
- [28] Weber, M., Generic morphisms, parametric representations and weakly Cartesian monads, *Theory Appl. Categ.*, 13, 191-234, (2004) · [Zbl 1062.18008](#)
- [29] Weber, M., Familial 2-functors and parametric right adjoints, *Theory Appl. Categ.*, 18, 665-732, (2007) · [Zbl 1152.18005](#)

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.