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Division algebras and supersymmetry. I. (English) Zbl 1210.81117

Doran, Robert S. (ed.) et al., Superstrings, geometry, topology, and C^*-algebras. Collected papers based on the presentations at the NDF-CBMS regional conference on mathematics on topology, C^*-algebras, and string duality, Fort Worth, TX, USA, May 18–22, 2009. Providence, RI: American Mathematical Society (AMS) (ISBN 978-0-8218-4887-6/hbk). Proceedings of Symposia in Pure Mathematics 81, 65-80 (2010).

It is well known that the theory of a Yang-Mills field coupled to a massless spinor transforming in the adjoint representation of the gauge group is supersymmetric iff the dimension of spacetime is $3, 4, 6, 10$. This paper aims to present a self-contained proof of its "if" part, exploiting the theory of normed division algebras. The result can be traced back to [Nucl. Phys., B 121, 77–92 (1977)]. Evans [Nucl. Physics, B 298, 92–108 (1988)] has shown that the supersymmetry in the above theory in dimension $n + 2$ implies the existence of a normed division algebra of dimension n, while Kugo and Townsend [Nucl. Phys., B 221, 357–380 (1983)] have established its converse by showing how spinors in dimension $3, 4, 6, 10$ inherit special properties from the normed division algebras $\mathbb{R}$ (the real numbers), $\mathbb{C}$ (the complex numbers), $\mathbb{H}$ (the quaternions) and $\mathbb{O}$ (the octonians). *A. Sudbery* [J. Phys. A 17, 939–955 (1984; Zbl 0544.22010)] has constructed vectors, spinors and Lorentz groups in Minkowski spacetimes of dimensions $3, 4, 6, 10$ by using division algebras, which was followed by an octonionic proof of the supersymmetry in dimension 10 in [Phys. Lett., B 198, 161–164 (1987)] and [Phys. Rev. D 12, 4073–4077 (1989)]. The idea was simplified in [*T. Dray, J. Janesky* and *C. A. Manogue*, Adv. Appl. Clifford Algebr. 10, No. 2, 193–216 (2000; Zbl 0984.15007)], while *J. Schray* [Classical Quantum Gravity 13, No. 1, 27–38 (1996; Zbl 0999.81500)] applied the same machinery to the superparticle.

For the entire collection see [Zbl 1201.81004].

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MSC:

81T60	Supersymmetric field theories
81R25	Spinor and twistor methods in quantum theory
81Q60	Supersymmetry and quantum mechanics
11R52	Quaternion and other division algebras: arithmetic, zeta functions
15B33	Matrices over special rings (quaternions, finite fields, etc.)
46S10	Functional analysis over fields (not $\mathbb{R}$, $\mathbb{C}$, or $\mathbb{H}$)
81T13	Yang-Mills and other gauge theories

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