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Quantum theory for mathematicians. (English) Zbl 1273.81001

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The mathematical culture and the physical culture are very near and very remote at the same time. This ambivalence is often irritating to both mathematicians and physicists. As is well known, Paul A. M. Dirac believed that, if there is a God, he is a great mathematician. Eugene P. Wigner has stated that the miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve. Even in ancient times, Euclid held that the laws of Nature are but the mathematical thoughts of God. Nevertheless, it is not easy for a general mathematician to study physics, in particular, quantum theory. Books on quantum mechanics written by physicists are not subject to the high-level precision or exactness that mathematicians are accustomed to. Physicists often use different terminology and notation for familiar mathematical concepts, which are terribly confusing to hasty mathematicians. Although mathematicians are keenly aware of the great influence of quantum physics upon mathematics such as the Wigner-Mackey theory of induced representations, the Kirillov-Kostant orbit method and quantum groups, mathematicians who want to study quantum mechanics for the first time should choose a textbook truly adequate for them so as to avoid spending a lot of time in vain.

There are a few textbooks on quantum theory for mathematicians who are alien to the physical culture, say, Zbl 1156.81004; Zbl 0932.81001; Zbl 1140.81001; Zbl 1200.81001; Zbl 1275.81003; Zbl 1275.81002; Zbl 1155.81003; Zbl 1200.81001; Zbl 1221.81004; Zbl 0984.00503; Zbl 1124.81002; Zbl 1155.81005; Zbl 1228.81005; Zbl 1166.81004, and so on, but this modest textbook will surely find its place. All in all, the book is well written and accessible to any interested mathematicians and mathematical graduates.

The organization of the book goes as follows. Chapter 1 is devoted to the historical origins of quantum theory. Chapter 2 is a hasty introduction to classical mechanics, including a short treatment of Poisson brackets and Hamilton's form of Newton's equations. The Hamiltonian approach to classical mechanics is generalized to manifolds in Chapter 21. Chapter 3 is concerned with the essentially probabilitistic nature of quantum theory, where the probabilities are captured by the position and momentum operators while the time-evolution of the wave equation is described by the Hamiltonian operator. Chapter 4 deals with several methods of solving the free Schrödinger equation in dimension one, while Chapter 5 is concerned with a particle in a square well. The author spends the succeeding five chapters on the spectral theorem. After giving perspectives on the theorem in Chapter 6, the spectral theorem for bounded self-adjoint operators is stated in Chapter 7 and established in Chapter 8. The spectral theorem for unbounded self-adjoint operators is stated and proved in terms of projection-valued measures in Chapter 10 after an introduction to unbounded self-adjoint operators in Chapter 9. The succeeding four chapters are concerned with the canonical commutation relations in some way or other. Chapter 11 deals with the harmonic oscillator, introducing the algebraic approach to quantum mechanics in place of analysis as the way to solve quantum systems. Harmonic oscillators are rampant in physics. The Heisenberg uncertainty principle and the Stone-von Neumann theorem are two important consequences of the commutation relations between the position and momentum operators, the former being dealt with in Chapter 12 and the latter being dealt with in Chapter 14. Both results are examined carefully with due regard to certain technical domain conditions. Due to Groenewold's no-go theorem [Zbl 0060.45002], there is no single perfect quantization scheme, but the Weyl quantization, on which the author spends a large portion of Chapter 13, is regarded as having the best properties. The succeeding four chapters are concerned with less elementary properties of quantum theory. Chapter 15 is devoted to so-called WKB (standing for Gregor Wentzel, Hendrik Kramers and Léon Brillouin) approximation, which gives an approximation to the eigenfunctions and eigenvalues of the Hamiltonian operator in one dimension. Chapter 16 gives a brief introduction to Lie groups, Lie algebras and their representations, which is put to use in Chapter 17 for studying angular momentum and spin in terms of representation of $\mathbf{SO}(3)$, where it is noted that the notion of fractional spin is to be understood as a representation of the Lie algebra of $\mathbf{SO}(3)$, which has no corresponding representation of $\mathbf{SO}(3)$ itself. I note gladly in passing that the author is the author of a successful book on Lie groups and Lie algebras [Zbl 1026.22001]. Chapter 18 is devoted to describing the energy levels of the hydrogen atom, including some discussion on its hidden symmetries. Chapter 19 is concerned with composite systems in terms of tensor products of Hilbert spaces, bosons and fermions, the Pauli exclusion principle, and so on. The coda of the book consists of three chapters, dealing with some advanced topics on classical and quantum mechanics. Chapter 20 develops the path integral formulation of quantum mechanics rigorously by using the Wiener measure. The chapter begins with the Trotter product formula, then turning to the heuristic formulation of Feyman himself and finally obtaining the so-called Feyman-Kac formula. Chapter 22 considers the geometric quantization program from a symplectic viewpoint, paving the way to Chapter 23, which lays out in an orderly fashion all the ingredients (bundles, connections, polarizations, etc.) needed to do geometric quantization generally.

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

81-01	Textbooks (quantum theory)
00A79	Physics
81Qxx	General mathematical topics and methods in quantum theory
81Rxx	Groups and algebras in quantum theory
81Sxx	General quantum mechanics and problems of quantization

Keywords:

quantum mechanics; spectral theorem; harmonic oscillator; Heisenberg's uncertainty principle; Schrödinger equation; Weyl quantization; path integral; Wiener measure; Trotter product formula; Feyman-Kac formula; geometric quantization; unbounded self-adjoint operator; Pauli exclusion principle

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