**Dotsenko, Vladimir**; **Shadrin, Sergey**; **Vallette, Bruno**
**Toric varieties of Loday's associahedra and noncommutative cohomological field theories.**
(English) Zbl 1421.18002
J. Topol. 12, No. 2, 463-535 (2019).

The principal objective in this paper is to define analogues of the little 2-discs operad, the framed little 2-discs operad and the Deligne-Mumford operad that ultimately lead to a noncommutative analogue of the notion of a cohomological field theory.

The paper consists of 9 sections together with an appendix on brick manifolds and Loday polytopes. §2 recalls some definitions used throughout the paper.

§3 proposes noncommutative versions of the topological operads of little 2-discs and framed little 2-discs leading respectively to the algebraic operads of noncommutative Gerstenhaber and noncommutative Batalin-Vilkovisky algebras which control the algebraic structures studied in [*K. Borjeson*, J. Gen. Lie Theory Appl. 9, No. 1, Article ID 1000214, 5 p. (2015; Zbl 1357.16022); *V. Ginzburg* and *T. Schedler*, Adv. Math. 231, No. 3–4, 2352–2389 (2012; Zbl 1356.16008)].

§4 adapts various definitions and results in [*G. C. Drummond-Cole* et al., J. Homotopy Relat. Struct. 5, No. 1, 15–36 (2010; Zbl 1280.18009); *I. Gálvez-Carrillo* et al., J. Noncommut. Geom. 6, No. 3, 539–602 (2012; Zbl 1258.18005); *E. Getzler*, Commun. Math. Phys. 163, No. 3, 473–489 (1994; Zbl 0806.53073); *E. Getzler*, Prog. Math. 129, 199–230 (1995; Zbl 0851.18005)] to the noncommutative case; the operad of noncommutative hypercommutative algebras, or noncommutative cohomological field theories arises, within this setting, from homotopy theory of noncommutative Batalin-Vilkovisky algebras, whose minimal model is computed.

§5 presents two geometric constructions of a nonsymmetric version of the operad $\left\{\overline{\mathcal{M}}_{0,n+1}(k)\right\}$ of the moduli spaces of stable complex curves with marked points making use of toric varieties and the theory of wonderful models of subspace arrangements.

§6 presents a somewhat elegant description of the intersection product in the cohomology rings of complex brick manifolds, which is particularly useful for defining the Givental group action on noncommutaive hypercommutative algebras.

§7 addresses two applications of the Givental action. §8 establishes that the noncommutative analogues of the operads of the little 2-discs, the framed little 2-discs and the Deligne-Mumford spaces are formal.

§9 explains geometrically some relations in the rational homology of the real brick operad. The appendix demonstrates that under the general correspondence between the toric varieties, fans and polytopes, the polytope associated to the brick manifold $\mathcal{B}(\underline{n})$ is the Loday polytope $L_n$ realizing the Stasheff polytope.

Reviewer: Hirokazu Nishimura (Tsukuba)

**MSC:**

| | |
|---|---|
| 18D50 | Operads |
| 14M25 | Toric varieties, Newton polyhedra |
| 53D45 | Gromov-Witten invariants, quantum cohomology, Frobenius manifolds |
| 81T45 | Topological field theories |

**Full Text:** DOI