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A distance exponent for Liouville quantum gravity. (English) Zbl 07049484
Probab. Theory Relat. Fields 173, No. 3-4, 931-997 (2019).

Let $\gamma \in (0,2)$ and let $D \subset \mathbb{C}$ be a simply connected domain. A *γ-Lioubille quantum gravity (LQG) surface* is the surface parametrized by D whose Riemannian metric tensor is given by

$$e^{\gamma h}\left(dx^2+dy^2\right)$$

where h is some variant of the Gaussian free field [*S. Sheffield*, Probab. Theory Relat. Fields 139, No. 3–4, 521–541 (2007; Zbl 1132.60072)] on D, dx^2+dy^2 is the Euclidean metric tensor, and d_γ is the so-called dimension of γ-LQG. One major problem in the study of LQG is to make sense of the formula as a random metric. Another majoir problem is to determine the Hausdorff dimension of the γ-LQG metric under the assumption that it exists.

The principal objective in this paper is to give some progress toward the above two problems by making use of the peanosphere construction [“Liouville quantum gravity as a mating of trees”, `arXiv:1409.7055`]. A *peanosphere* is a random pair (M,η) consisting of a topological space M and a space-filling curve γ on M (with a specified parametrization) constructed from a correlated two-sided two-dimensional Brownian motion. It is shown in [arXiv:1409.7055] that there is a canonical, up to rotation, embedding of a peanosphere into $\mathbb{C}$ such that the peanosphere volume measure is mapped into the γ-quantum area measure corresponding to a particular type of γ-LQG surface $(\mathbb{C},h,0,\infty)$ called a *γ-quantum cone* there. This article studies a family of planar maps $\{\mathcal{G}^\epsilon\}_{\epsilon>0}$, called the *LQG structure graphs* (also called *mated-CRT maps*) associated with the pair (h,η), which is expected to converge in the scaling limit in the Gromov-Hausdorff sense (when equipped with their graph distances) to an LQG metric induced by h.

The first main result of the paper (Theorem 1.10) gives upper and lower bounds for the scaling dimension of $\mathcal{G}^\epsilon$. The second main result of the paper (Theorem 1.12) is the existence of an exponent χ for distances in the LQG structure graphs. The third main result of the paper (Theorem 1.15) is estimates the probability that distances between vertices of $\mathcal{G}^\epsilon\mid_{(0,1]}$ are of order $\varepsilon^{-\chi+o_\varepsilon(1)}$ without stating that the diameter of $\mathcal{G}^\epsilon\mid_{(0,1]}$ is at least $\varepsilon^{-\chi+o_\varepsilon(1)}$.

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MSC:

83C45	Quantization of the gravitational field
60J67	Stochastic (Schramm-)Loewner evolution (SLE)
60D05	Geometric probability and stochastic geometry
60J65	Brownian motion

Full Text: DOI

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