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Notes on super Riemann surfaces and their moduli. (English) Zbl 1423.32012

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The principal objective in this paper is to give a relatively understandable account of portion of super Riemann surface theory that is relevant for superstring perturbation theory [the author, Pure Appl. Math. Q. 15, No. 1, 213–516 (2019; Zbl 1421.81101)]. The material described in this paper is fairly standard with the exception that no relation between holomorphic and antiholomorphic odd variables is assumed, which would be unnatural for superstrings. A companion paper [the author, Pure Appl. Math. Q. 15, No. 1, 3–56 (2019; Zbl 1421.58001)] gives an introduction to supermanifolds and integration.

The paper consists of 9 sections. §2 gives an introduction to super Riemann surfaces from a purely holomorphic standpoint, while §3 addresses them from a smooth viewpoint. §4 revisits some of the considerations in §2 in the presence of Ramond punctures. Some examples of low genus are discussed in §5. §6 describes the behavior of supermoduli space at infinity. The moduli space $\mathfrak{M}$ of super Riemann surfaces is an analogous Deligne-Mumford compactification $\widehat{\mathfrak{M}}$ obtained by adjoining to $\mathfrak{M}$ certain divisors $\mathcal{D}_\lambda$ parametrizing super Riemann surfaces with nodes. As in the bosonic strings, the subtleties associated to the behavior at infinity are infrared effects, but, different from the bosonic strings, the infrared effects are completely harmless in the superstring case, where there are no tachyon poles, and the massless particle tadpoles are tamed by superspace supersymmetry. The infrared behavior is the same as one would expect in a field theory with the same low evergy content. §7 considers *unorientable* super Riemann surfaces *with boundary*, while, up to §6, only orientable super Riemann surfaces without boundary are addressed. Type I superstring theory and more general orientifolds of Type II superstring theory are concerned with unorientable strings. Open as well as closed strings appear in Type I superstring theory and more generally in Type II superstring theory in the presence of D-branes. As is well known, when open strings are present, the string worldsheet is a super Riemann surface with boundary. Therefore, in Type I superstring theory and in more general Type II arenas with both orientifolds and D-branes, super Riemann surfaces simultaneously unorientable and with boundary should be considered. The goal in §8 is to discuss the analogue on a super Riemann surface of the periods of a holomorphic differential on an ordinary Riemann surface. §9 describes some pretty facts about $\mathcal{N}=2$ super Riemann surfaces, complex supermanifolds of dimension 1/1, and duality. Three appendices on Berezinians, pictures and picture-changing in the presence of a Ramond divisor are accompanied.

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