

**López Franco, Ignacio**

**Cofibrantly generated lax orthogonal factorisation systems.** (English) Zbl 07131196  
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Lax orthogonal factorization systems were introduced in [*M. M. Clementino* and *I. López Franco*, Adv. Math. 302, 458–528 (2016; [Zbl 1375.18032](#))] as a type of algebraic weak factorization systems on 2-categories. This paper is concerned with the cofibrant generation of lax orthogonal factorization systems (called cofibrant KZ-generation), while the cofibrant generation of algebraic weak factorization systems [*R. Garner*, Appl. Categ. Struct. 17, No. 3, 247–285 (2009; [Zbl 1173.55009](#)); *ibid.* 20, No. 2, 103–141 (2012; [Zbl 1256.55005](#))] on locally presentable categories was investigated in [*J. Bourke* and *R. Garner*, J. Pure Appl. Algebra 220, No. 1, 108–147 (2016; [Zbl 1327.18004](#))], encompassing a plenty of examples but failing to reach such important ones as those based upon the category of topological spaces.

This paper aims to fill the gap in the literature. The author shows that cofibrantly KZ-generated algebraic weak factorization systems are always lax orthogonal factorization systems. Representable multicategories, which this paper studies in some detail, are not locally presentable categories. The author studies the case of categories enriched in posets and the cofibrant KZ-generation thereon, encompassing topological spaces. There are two ways of constructing new lax orthogonal factorization systems, namely, via simple adjunctions and via cofibrant KZ-generation, the two procedures of which are shown to yield distinct lax orthogonal factorization systems.

Reviewer: [Hirokazu Nishimura \(Tsukuba\)](#)

#### MSC:

- [18A32](#) Factorization of morphisms, substructures, etc.
- [55U35](#) Abstract homotopy theory; axiomatic homotopy theory
- [18D20](#) Enriched categories (over closed or monoidal categories)

#### Keywords:

[algebraic weak factorization system](#); [weak factorization system](#); [lax factorization system](#); [multicategory](#); [continuous lattice](#); [orthogonal factorization system](#); [cofibrant generation](#)

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