Bunge, Marta; **Gago, Felipe**; **San Luis Fernández, Ana María**
Synthetic differential topology. (English) ｜Zbl 1398.57002｜
London Mathematical Society Lecture Note Series 448. Cambridge: Cambridge University Press (ISBN 978-1-108-44723-2/pbk; 978-1-108-55349-0/ebook). ix, 223 p. (2018).

Formally speaking, Synthetic Differential Topology (SDT) is obtained from Synthetic Differential Geometry (SDG) by adding axioms of a local nature to SDG. While the basic axioms of SDG are concerned with the representability of jets of smooth mappings by tiny objects of an algebraic nature, those of SDT are engaged in the representability of germs of smooth mappings by tiny objects of a logical nature originating with [*J. Penon*, Diagrammes 13, 191 p. (1985; Zbl 0558.18003); Cah. Topologie Géom. Différ. Catégoriques 22, 67–72 (1981; Zbl 0463.18005)].

This first book on SDT is divided into six parts. Part I consists of Chapters 1 and 2, giving preliminaries on topos theory and synthetic differential geometry (SDG). Part II, concerned with topics in SDG, consists of Chapters 3 and 4. Chapter 3 addresses the Ambrose-Palais-Singer theorem, [*W. Ambrose* et al., Anais Acad. Brasil. Ci. 32, 163–178 (1960; Zbl 0097.37904)] in SDG. Chapter 4, based on [*M. Bunge* and *M. Heggie*, Contemp. Math. 30, 30–62 (1984; Zbl 0582.18006)], deals with the calculus of variations.

Part III consists of Chapters 5 and 6. Chapter 5 is concerned with the Euclidean and the weak topological structures [*M. Bunge* and *E. J. Dubuc*, Lect. Notes Pure Appl. Math. 106, 93–159 (1987; Zbl 0658.18004) and *M. Bunge* and *F. Gago*, J. Pure Appl. Algebra 55, No. 3, 213–250 (1988; Zbl 0657.18006)]. Chapter 6 gives axioms for SDT.

Part IV consists of Chapters 7 and 8. Chapter 7 deals with Mather's theorem within SDT [loc. cit. and *A. M. San Luis Fernández*, Estabilidad transversal de gérmenes representables infinitesimalmente (Spanish; Castilian). Santiago de Compostela: Universidad de Santiago de Compostela, Departamento de Álgebra (1997; Zbl 0938.58001)], enabling the authors to develop the theory of stability of germs of smooth functions axiomatically within SDT. Chapter 8 is an application of Mather's theorem to Morse theory within SDT [*F. Gago*, Lect. Notes Math. 1348, 125–129 (1988; Zbl 0661.18004); Internal weak opens, stability and Morse theory for synthetic germs. Santiago de Compostela: Universidad de Santiago de Compostela, Dep. de Algebra (1989; Zbl 0667.51001)] and Bull. Soc. Math. Belg., Sér. A 41, No. 2, 279–287 (1989; Zbl 0688.18007)].

Part V consists of Chapters 9 and 10. Chapter 9 extends the notion of a well adapted model of SDG to SDT. Chapter 10 addresses stability of unfoldings of germs as a particular case of stability of germs by taking advantage of the model theory developed in the previous chapter.

Part VI consists of Chapters 11 and 12. Chapter 11 recalls the construction of the Dubuc topos $\mathcal{G}$ by means of $\mathcal{C}^\infty$-rings and germ determined ideals [*E. J. Dubuc*, Am. J. Math. 103, 683–690 (1981; Zbl 0483.58003)]. Chapter 12 is devoted to establishing that $\mathcal{G}$ is indeed a model of SDT.

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MSC:

57-02	Research monographs (manifolds)
57R99	Differential topology
58A03	Topos-theoretic approach to differentiable manifolds
18F15	Abstract manifolds and fiber bundles
18B25	Topoi
51K10	Synthetic differential geometry

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