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Fuentes-Keuthan, Daniel; **Kędziorek, M.**; **Rovelli, M.**
A model structure on prederivators for $(\infty,1)$-categories. (English) Zbl 07130455
Theory Appl. Categ. 34, 1220-1245 (2019).

A *prederivator*, the notion of which was introduced independently in [https://webusers.imj-prg.fr/~georges.maltsiniotis/groth/Derivateurs.html], [*A. Heller*, Homotopy theories. Providence, RI: American Mathematical Society (AMS) (1988; Zbl 0643.55015)] and [http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.170.3817&rep=rep1&type=pdf], is a contravariant 2-functor

$$\mathbb{D} : \mathcal{C}at^{\mathrm{op}} \to \mathcal{C}AT$$

minimally recording the homotopical information of a given $(\infty,1)$-category. *O. Renaudin* [J. Pure Appl. Algebra 213, No. 10, 1916–1935 (2009; Zbl 1175.18008)] has established that prederivators carry all the relevant information of a given $(\infty,1)$-category.

Inspired by the classical theorem of Brown, *K. Carlson* ["On the ∞-categorical Whitehead theorem and the embedding of quasicategories in prederivators", arXiv:1612.06980] raised the question whether the essential image of $\mathbb{H}o$ could be characterized. This paper provides such a characterization (§2.16) by introducing the notion of quasi-representability (§2.8).

Theorem 1. A prederivator $\mathbb{D}$ is quasi-representable if and only if it is of the form

$$\mathbb{D} \cong \mathbb{H}o(X)$$

for some quasi-category X.

The authors then carry out homotopical analysis, identifying a suitable notion of weak equivalence of prederivators so that the homotopy category of the category $p\mathbb{D}er^{st}$ of small prederivators and strict natural transformations is equivalent to that of the category $q\mathcal{C}at$ of small quasicategories. The principal result in this paper (§3.6) is that this class of weak equivalences is part of a model structure on $p\mathbb{D}er^{st}$, which is equivalent to the model structure quasicategories. That is to say, we have

Theorem 2. There exists a cofibrantly generated model structure on the category $p\mathbb{D}er^{st}$ which is Quillen equivalent to the Joyal model structure on the category $s\mathcal{S}et$ of simplicial sets.

The desired model structure is transferred from the so-called Joyal model structure on $s\mathcal{S}et$ via a certain functor

$$R : p\mathbb{D}er^{st} \to s\mathcal{S}et$$

which was already used to establish [arXiv:1612.06980, Proposition 2.9]. The authors intend to give a rigorous comparison between the category theory of prederivators and that of quasicategories in future.

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

55U35 Abstract homotopy theory; axiomatic homotopy theory
18G30 Simplicial sets; simplicial objects in a category
18A25 Functor categories, comma categories

Keywords:
prederivator; model structure; $(\infty,1)$-category; quasi-category

Full Text: Link

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