Vickers, Steven

Sketches for arithmetic universes. (English) Zbl 1420.18011

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The author set out a platform of using Joyal's arithmetic universes (AUs) to provide a uniform, base-independent setting for geometric reasoning about toposes as generalized spaces [Math. Struct. Comput. Sci. 9, No. 5, 569–616 (1999; Zbl 0946.18001)]. The aim of the present paper is to give a formal basis for this purpose, while a companion paper [Cah. Topol. Géom. Différ. Catég. 58, No. 3–4, 213–248 (2017; Zbl 1398.18004)] investigates in greater depth the relation with toposes. The main accomplishment of this paper (§8) is the construction of a 2-category $\mathfrak{Con}$ serving as one of generalized spaces, which is analogous to the 2-category $\mathfrak{Top}$ of Grothendieck toposes and geometric morphisms, or, more generally, to the 2-category of bounded $\mathcal{S}$-toposes with $\mathcal{S}$ a chosen base elementary topos with nno. It is shown (Theorem 51) that a 2-functor from $\mathfrak{Con}$ to AUs and strict AU-functors, mapping presentations $\mathbb{T}$ to the presented AUs $\mathbf{AU}\langle\mathbb{T}\rangle$, is full and faithful, meaning that the 1-cells and 2-cells have been defined in a sufficiently general manner.

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References: