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A_∞-algebra from supermanifolds. (English) Zbl 07125531

Ann. Henri Poincaré 20, No. 12, 4163-4195 (2019).

The study of supermanifolds and their peculiar geometry attracts renewed interest mainly because of superstring field theory [*E. Witten*, Pure Appl. Math. Q. 15, No. 1, 517–607 (2019; Zbl 1421.81102); ibid. 15, No. 1, 213–516 (2019; Zbl 1421.81101); ibid. 15, No. 1, 57–211 (2019; Zbl 1423.32012); ibid. 15, No. 1, 3–56 (2019; Zbl 1421.58001)].

There are two ways to construct the supersymmetric sigma model representing the perturbative Lagrangian of strings, namely, the Ramond-Neveu-Schwarz (RNS) formulation and the Green-Schwarz/Pure Spinor (GS/PS) one, both of which require the insertion of picture changing operators to cancel the anomalies and to make the amplitudes meaningful. Both the cases of picture changing operators are to be understood from a genuinely geometrical viewpoint by having a meaningful integration theory for supermanifolds, which in turn requires deep understanding of the occurences of peculiarities whenever part of the geometry is anticommuting. It turns out in particular that a new number (called *picture number*) counting the number of delta functions of the differential 1-forms (i.e., $\delta(d\theta_i)$ is needed to describe forms on a supermanifold. Working on a supermanifold of dimension n, forms of degree n and picture p are really sections of the Berezinian sheaf, and they can be integrated over. Besides the usual differential operator d, a new operator η emerges, corresponding physically to the embedding of the $N = 1$ RNS superstring into a $N = 4$ supersymmetric sigma model [*N. Berkovits* and *C. Vafa*, Nucl. Phys., B 433, No. 1, 123–180 (1995; Zbl 1020.81761)], as well as picture changing operators Y and Z are needed.

Over the years, there have been several proposed actions reproducing the full-fledged superstring spectrum where the insertion of picture changing operators is crucial, though none of them turned out to be fully consistent. To avoid this problem, new multilinear operations forming a non-associative algebra (called A_∞-algebra) have been proposed [*T. Erler* et al., J. High Energy Phys. 2014, No. 4, Paper No. 150, 30 p. (2014; Zbl 1333.81326)]. The principal objective in this paper is to elaborated the construction of A_∞-algebra by introducing some multilinear products. Mimicking what has been done for string field theory while using now ingredients that arise from the geometry of supermanifolds, the authors construct multilinear products of forms, some of which have precisely the same form of those coming from superstring field theory (constructed in terms of wedge products and picture changing operators insertion while the richness emerging from the geometry of forms on supermanifolds leads to new products, turning the exterior algebra of forms into a non-associative algebra which might lead to a generalization of the above-mentioned A_∞-algebra construction. Their originality lies in multiple ferminonic directions.

This paper, consisting of five sections and two appendices, discusses the differential operators d and η in §3 after a review of the basics in the theory of forms on supermanifolds in §2. §4 gives a review of a useful construction of the picture changing operator Z [Nucl. Phys. B 899, 570 (2015)]. §5 introduces and discusses the A_∞-algebra of forms on a supermanifold for the case of one fermionic directions, and provides the definition of multiproducts for higher dimensions. Appendix A gives mathematical foundations of A_∞-algebra via cotensor algebra and coderivations, while Appendix B is engaged in useful computations.

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MSC:

58A General theory of differentiable manifolds
58A12 de Rham theory (global analysis)
58A50 Supermanifolds, etc. (global analysis)

Full Text: DOI