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Hypergraph categories. (English) Zbl 1422.18010

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Hypergraph categories were rediscovered several times in several guises with several names, including well-supported compact closed categories, dgs-monoidal categories and dungeon categories. The resurgent nature of this notion comes from the reason that it has a lot of application, including automata theory, databases, circuits, graph rewriting and belief propagation, and besides, the standard definition is too involved and too ornate to grasp immediately. Here a *hypergraph category* is a symmetric monoidal category in which every object is equipped with the structure of a special commutative Frobenius monoid obeying certain compatibility with the monoid product. The principal objective in this paper is to show precisely that a hypergraph category is simply a cospan-algebra, which is, roughly speaking, a lax monoidal from cospans to sets. It was demonstrated in [*D. I. Spivak* et al., J. Pure Appl. Algebra 221, No. 8, 2064–2110 (2017; Zbl 06817576)] that the operad governing traced monoidal categories is the operad **Cob** of oriented 1-dimensional cobordisms. It is similarly shown in this paper that the operad governing hypergraph categories is **Cospan**, meaning informally that there is a one-to-one correspondence between the wiring diagrams interpretable in a hypergraph category $\mathcal{H}$ and cospans labelled by the objects of $\mathcal{H}$. The authors think of the cospan representation as an unbiased viewpoint on hypergraph categories, considering the category of cospan-algebras as a decategorification of the 2-category $\mathcal{H}yp$ of hypergraph categories, which is the first main result in this paper. The second main result in this paper is the isomorphism of 1-categories

$$\mathbf{Hyp}_{\mathrm{OF}} = \int^{\Lambda \in \mathbf{Set}_{\mathrm{List}}} \mathbf{Lax}\left(\mathbf{Cospan}_{\Lambda}, \mathbf{Set}\right)$$

where OF stands for “objectwise-free”.

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18D50 Operads

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