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The tangent bundle of a model category. (English) Zbl 07121365

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This paper addresses the homotopy theory of the abstract cotangent complex and Quillen cohomology within the setting of model categories. The principal objective in this paper is to develop model-categorical gadgets in order to investigate the ∞ -categorical cotangent complex platform of *J. Lurie* [Higher Algebra, <http://www.math.harvard.edu/~char126/relaxlurie/papers/HA.pdf>, §7.3]. The authors construct a model-categorical setup which is exploited in their subsequent papers [High. Struct. 3, 17–66 (2019; Zbl 1418.55005), J. Topol. 11, No. 3, 752–798 (2018; Zbl 1405.55008) and “Tangent categories of algebras over operads”, Preprint, [arXiv:1612.02607](https://arxiv.org/abs/1612.02607)]. Their framework provides point-set ways to study the cotangent complex and Quillen cohomology making use of tools from homotopical algebra.

The paper consists of three sections. §2 starts with describing a model $\mathrm{Sp}(\mathcal{M})$ for the stabilization of a model category \mathcal{M} , which, applied to pointed objects in $\mathcal{M}/_A$, gives rise to the tangent model category $\mathcal{T}_A\mathcal{M}$ at A . It is described how the usual machinery of suspension- and Ω -spectrum replacements arises. §3 uses a similar approach to construct a model $\pi : \mathcal{T}\mathcal{M} \rightarrow \mathcal{M}$ for the tangent bundle of \mathcal{M} . It is shown (Theorem 3.25) that this provides a presentation of the ∞ -categorical projection

$$\int_{A \in \mathcal{M}_\infty} \mathcal{T}_A \mathcal{M}_\infty \rightarrow \mathcal{M}_\infty$$

where fibers are the tangent ∞ -categories of the ∞ -category \mathcal{M}_∞ underlying \mathcal{M} . The main result is that $\mathcal{T}\mathcal{M}$ is exhibited as a relative model category over \mathcal{M} in the sense of [*Y. Harpaz and M. Prasma*, Adv. Math. 281, 1306–1363 (2015; Zbl 1333.18024)] and forms a model fibration when restricted to the fibrant objects in \mathcal{M} . Furthermore, it is shown that when \mathcal{M} is tensored over a suitable model category \mathcal{S} , the tangent bundle $\mathcal{T}\mathcal{M}$ inherits the structure, becoming enriched over \mathcal{S} .

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MSC:

55P42 Stable homotopy theory, spectra
18G55 Nonabelian homotopical algebra
18D30 Fibered categories

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