Matsueda, Hiroaki

Quantum entanglements and their geometry. (Japanese) Zbl 1404.81002

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The principal objective in this book is to help understanding fundamental concepts in quantum entanglements and learn related techniques of mathematical physics. The reader will see the emerging unification of condensed matter physics, statistical physics, mathematical physics and string theory, which came from quantum information theory. The central riddle is how to unify, within the realm of mathematics, algebraic approaches exploited in information theory and quantum theory on the one hand and geometric approaches exploited in gravity theory on the other. Traditionally the interaction between information theory and theoretical physics has been unilateral, from physics to information theory (e.g., the application of the theory of spin glasses to problems in neural networks), but the recent development is the opposite direction, incorporating the techniques of information theory into physics. The author is an expert in condensed matter physics. The book consists of 10 chapters, Chapters 3–6 being concerned with algebraic approaches, Chapter 7 dealing with geometric approaches, while Chapters 9 and 10 presenting unified approaches.

Chapter 1, consisting of three sections, discusses the relationship between various arenas of physics and information theory, discussing from a standpoint of condensed matter physics (§1.1), from a standpoint of statistical physics and mathematical physics (§1.2) and from a standpoint of gravity theory (§1.3). The principal objective in Chapter 2 is to understand singular value decomposition and Suzuki-Trotter transformations. Chapter 3 addresses quantum entanglements, introducing entanglement entropy, discussing Bogoliubov transformations and studying entanglements in topological quantum systems. Chapter 4 is concerned with the proper construction of wave functions representing the scalability of quantum entanglements, investigating matrix product states in 1-dimension, addressing density matrix renormalization groups from a modern viewpoint and finally dealing with numerical optimization (generalized eigenvalue problems and time evolving brock decimation). Chapter 5 is concerned with tensor networks. Chapter 6 is concerned with the method of integrable systems. Chapter 7 is concerned with the relationship between information and gravity. It is very interesting to note that physics of black holes has accumulated analogous facts that physics of quantum entanglements has, for which to be scrutinized the knowledge of general relativity is indispensable. Chapter 7, consisting of 7 sections, addresses geometric aspects of black holes. §7.1 is concerned with metrics of spacetime and the scale law of entropy. §7.2 is a short course on hyperbolic geometry. §7.3 deals with curved spacetime. §7.4 addresses equations of gravity. §7.5 deals with Killing vector fields and anti-de Sitter spacetime. §7.6 addresses Schwarzschild black holes and BTZ (Maximo Bañados, Claudio Teitelboim and Jorge Zanelli) black holes. §7.7 is concerned with thermodynamics of black holes, discussing Bekenstein-Hawking law, Rindler spacetime, showing the equivalence of the first law of thermodynamics and the Einstein equation under the area law for entropy, investigating thermodynamic properties of quantum fields in Rindler spacetime and studying the mechanism of generating pairs of particles near horizons. Chapter 8 is concerned with conformal field theory and infinite-dimensional Lie algebras, beginning with Virasoro algebras, studying the Calabrese-Cardy formula, the finite χ scaling and the c theorem of Zamolodchikov. Chapter 9 is concerned with the correspondence by compactification of spacetime and the Ads/CFT (Anti-de Sitter/Conformal Field Theory) correspondence (also called the Maldacena duality or the gauge/gravity duality). The principal objective in Chapter 10 is to deepen our understanding of gauge/gravity duality by investigating geometry of memory spaces.

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

81-02 Research monographs (quantum theory)
81P40 Quantum coherence, entanglement, quantum correlations
81Q70 Differential-geometric methods in quantum mechanics
81T10 Model quantum field theories
82D20 Solids (statistical mechanics)
83C05 Einstein's equations (general structure, canonical formalism, Cauchy problems)
83C57 Black holes
00A79 Physics