Bunke, Ulrich

Homotopy theory with $*$-categories. (English) Zbl 1423.18011

Theory Appl. Categ. 34, 781-853 (2019).

A **-category* is a category with an involution * fixing the objects. This paper is concerned with the following categories $\mathcal{C}$ of *-categories and their marked versions $\mathcal{C}^+$:

1. *-categories ${}^*\mathbf{Cat}_1$: categories $\mathbf{A}$ endowed with an involution $*:\mathbf{A}\to\mathbf{A}^{\mathrm{op}}$.
2. $\mathbb{C}$-linear *-categories ${}^*_{\mathbb{C}}\mathbf{Cat}_1$: *-categories enriched over $\mathbb{C}$-vector spaces with an anti-linear involution.
3. pre-C^*-categories $C^*_{\mathrm{pre}}\mathbf{Cat}_1$: $\mathbb{C}$-linear *-categories which admit a maximal C^*-completion.
4. C^*-categories $C^*\mathbf{Cat}_1$: pre-C^*-categories whose Hom-vector spaces are complete in the maximal norm.

C^*-categories have been investigated in [*I. Dell'Ambrogio*, Homology Homotopy Appl. 14, No. 2, 101–127 (2012; Zbl 1261.46067)]. Many arguments in this paper are merely modifications of that paper.

The paper consists of 14 sections. §2 introduces the notion of *-category and various $\mathbb{C}$-linear versions. §3 is concerned with adjunctions. §4 addresses the representability of the functors taking the sets objects, morphisms, unitary morphisms of a (marked) *-category in the respective cases. §5 introduces the ∞-categories of *-categories, $\mathbb{C}$-linear *-categories, pre-C^*-categories, C^*-categories and their marked versions by inverting unitary (or marked, respectively) equivalences. §6 is engaged in the tensor and power structure over groupoids. §7 considers the functor categories, serving as explicit fibrant resolutions in §13, from the arrow category $\widetilde{G}$ with a group G. §8 is concerned with completeness, cocompleteness and local presentability. §9 claims the main theorem on the model category structures. §10 shows that the model category structures depicted in the previous section are simplicial model categories. §11 demonstrates that the model category structure on any of ${}^*\mathbf{Cat}_1$, ${}^*_{\mathbb{C}}\mathbf{Cat}_1$, $C^*\mathbf{Cat}_1$ and their marked versions is cofibrantly generated.

Given a group G with BG

$$\mathrm{Hom}_{BG}(\mathrm{pt},\mathrm{pt})$$

regarded as a *-category in such a way that

$$g^*=g^{-1}$$

one of the principal objectives in this paper is to calculate the object

$$\lim_{BG}\ell_{BG}(\mathbf{A})$$

in $\mathcal{C}_\infty$ for $\mathbf{A}$ in $\mathbf{Fun}(BG,\mathcal{C})$, which is tantamount to providing an object $\mathbf{B}$ of $\mathcal{C}$ with equivalence

$$l(\mathbf{B})\simeq\lim_{BG}\ell_{BG}(\mathbf{A})$$

where $l:\mathcal{C}\to\mathcal{C}_\infty$ denotes the localization inverting the (marked) unitary equivalences. §12 provides a candidate for $\mathbf{B}$, denoted by $\widehat{\mathbf{A}}^G$, whose justification is given in §13 (Theorem 13.7). §14 is concerned with infinity-categorical G-orbits, where it is established (Theorem 14.6) that

$$\operatorname*{colim}_{BG}\ell_{BG}(\mathbf{C})\simeq(\mathbf{C}\#BG)$$

for any $\mathcal{C}$ of ${}^*\mathbf{Cat}_1$, ${}^*_{\mathbb{C}}\mathbf{Cat}_1$, $C^*\mathbf{Cat}_1$ and their marked versions with an object $\mathbf{C}$ in $\mathcal{C}$.

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

18C35 Accessible and locally presentable categories
55U35 Abstract homotopy theory; axiomatic homotopy theory

Keywords:

∗-categories; model categories; ∞-categories; limits and colimits

Full Text: Link

References:

[1] J. Ad´amek and J. Rosicky. Locally Presentable and Accessible Categories. London Math. Soc. Lecture Notes Series 189, 1994.

[2] U. Bunke and A. Engel. Homotopy theory with bornological coarse spaces. https://arxiv.org/abs/1607.03657, 2016.

[3] T. Beke. Sheafifiable homotopy model categories. Math. Proc. Cambridge Philos. Soc., 129(3):447-475, 2000. · Zbl 0964.55018

[4] U. Bunke, A. Engel, D. Kasprowski, and Ch. Winges. Homotopy theory with marked additive categories.https://arxiv.org/abs/1811.04007.

[5] D.-Ch. Cisinski. Images directes cohomologiques dans les cat´egories de mod‘eles. Annales Math´ematiques Blaise Pascal, pages 195-244, 2003.

[6] D.-Ch Cisinski.Higher categories and homotopical algebra, volume 180 of Cambridge studies in advanced mathematics.Cambridge University Press, 2019. Available online underhttp://www.mathematik.uni-regensburg.de/ cisinski/CatLR.pdf.

[7] I. Dell'Ambrogio. The unitary symmetric monoidal model category of small C∗-categories. Homology, Homotopy and Applications, Vol. 14 (2012), No. 2, pp.101-127, 04 2010.

[8] W. G. Dwyer and D. M. Kan. Function complexes in homotopical algebra. Topology, 19(4):427-440, 1980. HOMOTOPY THEORY WITH ∗-CATEGORIES853 · Zbl 0438.55011

[9] W.G. Dwyer and D.M. Kan. Simplicial localizations of categories. J. of Pure and Applied Algebra, 17:267-284, 1980. · Zbl 0485.18012

[10] J. F. Davis and W. L¨uck. Spaces over a Category and Assembly Maps in Isomorphism Conjectures in K- and L-Theory. K-Theory, 15:201-252, 1998. · Zbl 0921.19003

[11] D. Dugger. Combinatorial model categories have presentations. Adv. Math., 164(1):177-201, 2001. · Zbl 1001.18001

[12] V. Hinich. Dwyer-Kan localization revisited. Homology Homotopy Appl., 18(1):27- 48, 2016. · Zbl 1346.18024

[13] Ph. S. Hirschhorn. Model categories and their localizations, volume 99 of Mathematical Surveys and Monographs. American Mathematical Society, Providence, RI, 2003.

[14] M. Hovey. Model Categories. Mathematical Surveys and Monographs vol. 63. Amer. Math. Amer. Math.Soc., 1999. · Zbl 0909.55001

[15] M. Joachim. K-homology of C∗-categories and symmetric spectra representing K-homology. Math. Ann., 327:641-670, 2003. · Zbl 1048.19005

[16] A. Joyal. Model structure on †-categories.https://ncatlab.org/nlab/show/ dagger+category#ModelStructure, 2010.

[17] J. Lurie. Higher topos theory, volume 170 of Annals of Mathematics Studies. Princeton University Press, Princeton, NJ, 2009. · Zbl 1175.18001

[18] J. Lurie. Higher algebra.http://www.math.harvard.edu/~lurie/papers/HA. pdf, 2017.