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Differential geometry from a singularity theory viewpoint. (English) Zbl 1369.53004

Hackensack, NJ: World Scientific (ISBN 978-981-4590-44-0/hbk; 978-981-4590-46-4/ebook). xiii, 368 p. (2016).

This book gives a detailed picture of the theory of interaction between manifolds and the theory of caustics and wavefronts, which enables us to deduce a lot of geometric information about surfaces immersed in the Euclidean 3, 4 and 5-spaces as well as space-like surfaces in the Minkowski spacetime.

The book consists of ten chapters. Chapter 1 highlights how singularity theory is to be used not only to recover classical results on curves and surfaces in a simpler and more elegant way but also to make the rich and deep underlying concepts involved appear naturally. The reader is referred to [*M. P. do Carmo*, Differential geometry of curves and surfaces. Englewood Cliffs, N. J.: Prentice-Hall, Inc. (1976; Zbl 0326.53001); Revised and updated 2nd edition of the 1976 edition published by Prentice-Hall. Mineola, NY: Dover Publications (2016; Zbl 1352.53002)] for a detailed study of the differential geometry of curves and surfaces.

Chapter 2 considers some aspects of the extrinsic geometry of a submanifold M of dimension n of the Euclidean space $\mathbb{R}^{n+r}$ with $r \geq 1$. Chapter 3 gives some basic definitions and results of the theory on singularities of germs of smooth mappings, initiated in [*H. Whitney*, Ann. Math. (2) 62, 374–410 (1955; Zbl 0068.37101)], that are necessary in the later chapters.

The authors recommend books [*V. I. Arnol'd* et al., Singularities of differentiable maps. Volume I: The classification of critical points, caustics and wave fronts. Transl. from the Russian by Ian Porteous, ed. by V. I. Arnol'd. Boston-Basel-Stuttgart: Birkhäuser (1985; Zbl 0554.58001); Reprint of the 1985 hardback edition. Boston, MA: Birkhäuser (2012; Zbl 1290.58001)], [*Th. Bröcker*, Differentiable germs and catastrophes. Translated by L. Lander. London etc.: Cambridge University Press (1975; Zbl 0302.58006)], [*C. G. Gibson*, Singular points of smooth mappings. London, San Francisco, Melbourne: Pitman (1979; Zbl 0426.58001)], [*J. Martinet*, Singularites des fonctions et applications différentiables. 2me ed. corr. Rio de Janeiro: Pontifica Universidade Catolica (1977; Zbl 0389.58005); Singularities of smooth functions and maps. Transl. from the French by Carl P. Simon. Cambridge etc.: Cambridge University Press (1982; Zbl 0522.58006)] for beginners in singularity theory, the book [*J. W. Bruce* and *P. J. Giblin*, Curves and singularities. A geometrical introduction to singularity theory. Cambridge etc.: Cambridge University Press (1984; Zbl 0534.58008); 2nd ed. Cambridge: Cambridge University Press (1992; Zbl 0770.53002)] for its applications in geometry of curves, and the survey paper [*C. T. C. Wall*, Bull. Lond. Math. Soc. 13, 481–539 (1981; Zbl 0451.58009)] for the study of finite determinacy of map germs.

Chapter 4 investigates the concept of contact between submanifolds as a singularity theoretic tool for the study of differential geometry of submanifolds of $\mathbb{R}^n$. The general theory of contact between submanifolds of any dimension of a given manifold can be found in [*J. A. Montaldi*, Mich. Math. J. 33, 195–199 (1986; Zbl 0601.53007)].

The generic singularities occurring in caustics and wavefronts and the way they deform as the original front is deformed were described in [*V. M. Zakaljukin*, Funct. Anal. Appl. 10, 23–31 (1976; Zbl 0331.58007); translation from Funkts. Anal. Prilozh. 10, No. 1, 26–36 (1976); Arnol'd et al., loc. cit]. The theory was initiated in [*L. Hörmander*, Acta Math. 127, 79–183 (1971; Zbl 0212.46601)]. It has a lot of applications [*V. M. Zakalyukin*, Proc. Steklov Inst. Math. 209, 114–123 (1995; Zbl 0883.93008); translation from Tr. Mat. Inst. Steklova 209, 133–142 (1995)], [*V. I. Arnol'd*, Proc. Int. Congr. Math., Warszawa 1983, Vol. 1, 27–49 (1984; Zbl 0566.58004)], [*S. Izumiya*, J. Differ. Geom. 38, No. 3, 485–500 (1993; Zbl 0781.57016)], [*S. Izumiya*, Proc. R. Soc. Edinb., Sect. A, Math. 125, No. 3, 567–586 (1995; Zbl 0843.58067)], [*S. Izumiya* and *G. T. Kossioris*, J. Differ. Equations 118, No. 1, 166–193 (1995; Zbl 0837.35091); Bull. Sci. Math. 121, No. 8, 619–667 (1997; Zbl 0908.35078); Arch. Ration. Mech. Anal. 139, No. 3, 255–290 (1997; Zbl 0907.35082)], [*S. Izumiya* et al., Q. Appl. Math. 59, No. 2, 365–390 (2001; Zbl 1027.35137); J. Math. Phys. 44, No. 5, 2077–2093 (2003; Zbl 1062.83071); Proc. Lond. Math. Soc. (3) 86, No. 2, 485–512 (2003; Zbl 1041.58017)]. Chapter 5 applies this to some aspects of the extrinsic geometry of a submanifold of

Euclidean spaces.

Chapter 6 applies the results from the previous chapters to the study of the extrinsic geometry of surfaces embedded in the Euclidean 3-space $\mathbb{R}^3$. Considerations are restricted to the local singularities of the relevant germs of functions and mappings. Then, Chapter 7 is devoted to the extrinsic differential geometry of a surface M immersed in $\mathbb{R}^4$. In order to illustrate how the singularity techniques are applied to the analysis of the extrinsic geometry of surfaces in higher codimensions, Chapter 8 deals with the geometric properties associated to the contacts of surfaces with hyperplanes and hyperspheres in $\mathbb{R}^5$.

The geometrical properties of submanifolds of the Minkowski space are investigated in a similar way to those of Euclidean spaces through the analysis of their contact with model submanifolds invariant by the Lorentz transformation group. Submanifolds can be space-like, time-like or light-like and the models can be the light-like hyperplanes or hyperspheres. The geometric properties of the contact of space-like submanifolds with space-like models do not differ radically from those of submanifolds of Euclidean spaces. A fascinating and geometrically rich situation arises in consideration of the contact of submanifolds with light-like hyperplanes or with lightcones. The authors call it *light-like geometry*, and Chapter 9 is devoted to it. It is to be seen as a generalization of *horospherical geometry*, which is concerned with the study of geometric properties derived from the contact of submanifolds with hyperhorospheres in hyperbolic space.

Chapter 10 shows how the techniques of singularity theory can be used to obtain global results on submanifolds M of Euclidean spaces or the Minkowski spacetime. The authors make use of three approaches. The first uses a stratification of the parameter space of the family of functions and mappings defined on M [*C. G. Gibson* et al., Topological stability of smooth mappings. Berlin-Heidelberg-New York: Springer-Verlag (1976; Zbl 0377.58006)]. The second, which is topological, computes the Euler characteristic of a surface either in terms of the total curvature or in terms of the number of certain stable singularity types of the members of the families of functions and maps on the surface. The third centers around the Poincaré-Hopf formula.

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

53-02	Research monographs (differential geometry)
53A05	Surfaces in Euclidean space
53C42	Immersions (differential geometry)
58K05	Critical points of functions and mappings
53A35	Non-Euclidean differential geometry

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