Shaumbwa, Vaino Tuhafeni

A characterization of the Higgins commutator. (English) Zbl 1420.18007

Appl. Categ. Struct. 27, No. 2, 159-162 (2019).

The Higgins commutator [*P. J. Higgins*, Proc. Lond. Math. Soc. (3) 6, 366–416 (1956; Zbl 0073.01704)] is a universal-algebraic generalization of the commutator of groups to Ω-groups. The paper [*S. Mantovani* and *G. Metere*, J. Algebra 324, No. 9, 2568–2588 (2010; Zbl 1218.18001)] claims that the Higgins commutator can be defined categorically for a pair of subobjects of each object in an ideal-determined category, while the paper [*M. Gran* et al., J. Algebra 397, 643–665 (2014; Zbl 1305.18040)] shows that the categorical definition of the Higgins commutator is to be recovered through the weighted subobject commutator. This paper aims at characterizing the Higgins commutator as the largest binary operation C defined on all subobjects of each object in an ideal-determined unital category, pursuant to the following conditions:

1. C is order-preserving.
2. For each pair of subobjects H and K of an object X, $C(H,K)$ is always less or equal to the meet of normal closures of H and K.
3. For every pair of subobjects H and K of an object X and every morphism f with domain X, we have
$$C(f(H), f(K)) = f(C(H,K))$$

A similar characterization is to be found in [*V. T. Shaumbwa*, Theory Appl. Categ. 32, 1588–1600 (2017; Zbl 1397.18010)] for the commutator introduced in [*S. A. Huq*, Q. J. Math., Oxf. II. Ser. 19, 363–389 (1968; Zbl 0165.03301)] within genuinely categorical setting. The paper [*J. Hagemann* and *C. Herrmann*, Arch. Math. 32, 234–245 (1979; Zbl 0419.08001)] gives a characterization of the commutator, introduced in [*J. D. H. Smith*, Mal'cev varieties. Cham: Springer (1976; Zbl 0344.08002)] and refined categorically in [*M. C. Pedicchio*, J. Algebra 177, No. 3, 647–657 (1995; Zbl 0843.08004)], on congruence lattices.

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MSC:

18B30 Categories of topological spaces and continuous mappings
18A05 Basic definitions of category theory
18B35 Preorders, orders and lattices (viewed as categories)
18A32 Factorization of morphisms, substructures, etc.

Keywords:

Higgins commutator; normal category; ideal-determined category; unital category

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