Date 6/21

$f: \mathbb{R}^n \to \mathbb{R}^m$

$x \in \mathbb{R}^n$で微分

$f'(x)\ \mathbb{R}^n \to \mathbb{R}^m$ 線形代数

$f' : \mathbb{R}^n \to \boxed{L(\mathbb{R}^n ; \mathbb{R}^m)}$ ($m \times n$の行列)

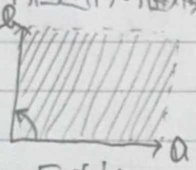
$\mathbb{R}^n$から$\mathbb{R}^m$への線型写像の全体 足し算

第9回

線型代数
線型写像
行列式

$S(a, b)$

平行四辺形の面積



aからbへ反時計回り

(1) $S(b, a) = -S(a, b)$

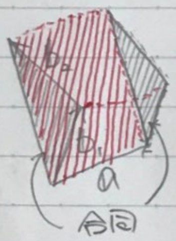
$S(a, a) = -S(a, a)$

(1) $2S(a, a) = 0 \qquad \alpha \in \mathbb{R}$

(2) $S(\alpha a, b) = \alpha S(a, b)$

$S(a, \beta b) = \beta S(a, b)$

(3) $S(a, b_1 + b_2) = S(a, b_1) + S(a, b_2)$



二重線型

$S(a_1 + a_2, b) = S(a_1, b) + S(a_2, b)$

(4) $e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$S(e_1, e_2) = 1$

$S(e_2, e_1) = -1$

$S(e_1, e_1) = S(e_2, e_2) = 0$

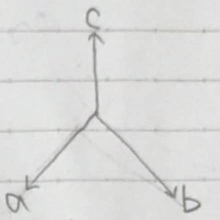
$a = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = a_1 e_1 + a_2 e_2$

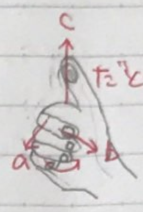
$b = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = b_1 e_1 + b_2 e_2$

$$\begin{aligned} S(a, b) &= S(a_1 e_1 + a_2 e_2, b_1 e_1 + b_2 e_2) \\ &= a_1 b_1 \underbrace{\boxed{S(e_1, e_1)}}_{(4)\text{より}\ 0} + a_1 b_2 \underbrace{\boxed{S(e_1, e_2)}}_{(4)\text{より}\ 1} \\ &\quad + a_2 b_1 \underbrace{\boxed{S(e_2, e_1)}}_{(4)\text{より}\ -1} + a_2 b_2 \underbrace{\boxed{S(e_2, e_2)}}_{(4)\text{より}\ 0} \\ &= a_1 b_2 - a_2 b_1 \end{aligned}$$

$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$

$a, b, c \in \mathbb{R}^3$





平行六面体

符号 + 親指側にあった
−

(1) $V(b, a, c) = -V(a, b, c)$

$V(c, b, a) = -V(a, b, c)$

どこか2つ入れかわると符号が変わる

$V(a, a, c) = 0$

(2) $V(\alpha a, b, c) = \alpha V(a, b, c)$

(3) $V(a_1 + a_2, b, c) = V(a_1, b, c) + V(a_2, b, c)$

(4) $V(e_1, e_2, e_3) = 1$

$\updownarrow$ 2つ入れかえた

$V(e_2, e_1, e_3) = V(e_3, e_2, e_1) = V(e_1, e_3, e_2) = -1$

三重線型

$V(e_2, e_3, e_1) = -V(e_1, e_3, e_2) = V(e_1, e_2, e_3)$

$V(e_1, e_2, e_3) = 0$

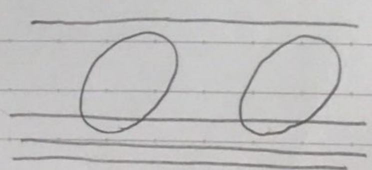
$a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = a_1 e_1 + a_2 e_2 + a_3 e_3$

$b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = b_1 e_1 + b_2 e_2 + b_3 e_3$

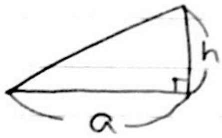
$c = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = c_1 e_1 + c_2 e_2 + c_3 e_3$

$V(a, b, c) = \cdots$ → report I $\qquad \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

covarieriの原理



底辺 : a
高さ : h



$\frac{1}{2}ah$

空間におけるCavalieriの原理