第8回

多変数の微分 $D=\{d\in\mathbb{R}\mid d^2=0\}$

Kock–Lawvereの公理

$\varphi: D\to\mathbb{R}$

$(\exists!\, a\in\mathbb{R})(\forall d\in D)$

$(\varphi(d)=\varphi(0)+ad)$

$f:\mathbb{R}\to\mathbb{R}\quad x\in\mathbb{R}$

$\varphi(d)=f(x+d)$

$\varphi(0)=f(x)$ } 一般化

$f(x+d)=f(x)+ad$

$\varphi: D\to\mathbb{R}^n$

$f(x)(\exists!\, a\in\mathbb{R}^n)(\forall d\in D)$

$(\varphi(d)=\varphi(0)+ad)$

$f:\mathbb{R}\to\mathbb{R}^n\quad x\in\mathbb{R}$

$\varphi(d)=f(x+d)$

$\varphi(0)=f(x)\quad(\exists!\, a\in\mathbb{R}^n)$

$f(x+d)=f(x)+ad$ → $f'(x)$

$f:\mathbb{R}^n\to\mathbb{R}^m$ を $x\in\mathbb{R}^n$ で微分する

$f'(x)=\mathbb{R}^n\to\mathbb{R}^m$ 線型写像

$a\in\mathbb{R}^n$

$f'(x)(a)\in\mathbb{R}^m$ これを決める

合成関数

$\varphi: t\in\mathbb{R}\mapsto x+at\in\mathbb{R}^n\quad \mathbb{R}\to\mathbb{R}^n$

$f\circ\varphi\ \ t\in\mathbb{R}\mapsto f(x+at)\quad \mathbb{R}\to\mathbb{R}^m$

$f'(x)$ $t=0$で微分

$\varphi'(t)=a\quad t\mapsto at$ (線形写像)

$\varphi'(0)=a$

$f'(\underbrace{x+a0}_{x})\circ\varphi'(0)=f'(x)(a)$

$f'(x)$

$f(x+ad)=f(x)+\overset{\mathbb{R}^m}{\boxed{\ }}d$

$f'(x)(a)$と書く(約束)

(1) $f'(x)(a_1+a_2)=f'(x)(a_2)$

$(\forall d\in D)$(2) $f'(x)(\alpha a)=\alpha f'(x)(a)\quad(\alpha\in\mathbb{R})$

→ report

$f(x+(a_1+a_2)d)=f(x)+\boxed{f'(x)(a_1+a_2)}d\quad(\forall d\in D)$

ここで微分　ここで微分

$f(x+a_1d+a_2d)=f(x+a_1d)+f'(x+a_1d)(a_2)d$

ここで微分　ここで微分

$= f(x)+f'(x)(a_1)d+\{\varphi(x)+\varphi'(x)(a_1)d\}d$

$f(x)(a_2)d$

$=f(x)+\{\boxed{f'(x)(a_1)+f'(x)(a_2)}\}d$

$f'(x)\,.\ a=\begin{pmatrix}a_1\\ \vdots\\ a_n\end{pmatrix}=a_1e_1+\dots+a_ne_n$

$f:\mathbb{R}^n\to\mathbb{R}^m$

$x\in\mathbb{R}^n\quad a\in\mathbb{R}^n$

$f'(x)(a)=f'(x)(a_1e_1+\dots+a_ne_n)$

線型 $f'(x)(e_1)a_1+\dots+f'(x)(e_n)a_n$

これがどんなものか考える $\Rightarrow \frac{\partial f}{\partial x_1}(x)$

$e_1=\begin{pmatrix}1\\0\\ \vdots\\0\end{pmatrix},\ e_2=\begin{pmatrix}0\\1\\0\\ \vdots\end{pmatrix}\quad x=\begin{pmatrix}x_1\\ \vdots\\ x_n\end{pmatrix}$

$f(x+e_1d)=f(x)+f'(x)(e_1)d$

$f\left(x+\begin{pmatrix}1\\0\\ \vdots\\0\end{pmatrix}d\right)=f(x)+?d$

$=f\begin{pmatrix}x_1+d\\x_2\\x_n\end{pmatrix}$

$=f\begin{pmatrix}x_1\\ \vdots\\x_n\end{pmatrix}+?d$

m×nの行列 $\mathbb{R}^m$

$\left[\frac{\partial f}{\partial x_1}(x),\ \frac{\partial f}{\partial x_2}(x),\ \dots,\ \frac{\partial f}{\partial x_n}(x)\right]\begin{pmatrix}a_1\\ \vdots\\ a_n\end{pmatrix}$

No.

Date

$f: \mathbb{R}^n \to \mathbb{R}^m$

$x \in \mathbb{R}^n$で微分

$f'(x)$ $\mathbb{R}^n \to \mathbb{R}^m$ 線型代数

$f': \mathbb{R}^n \to \boxed{L(\mathbb{R}^n; \mathbb{R}^m)}$ ($m \times n$の行列)

$\mathbb{R}^n$から$\mathbb{R}^m$への線型

写像の全体 足し算