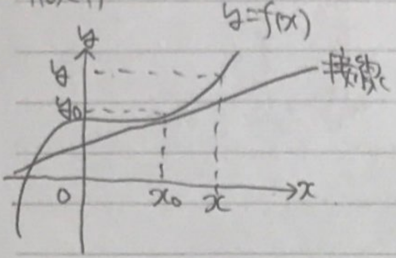
Date

微分



$\Delta x \longmapsto \Delta y$

$x - x_0 = \Delta x$

$y - y_0 = \Delta y$

$\Delta y = a \Delta x$

比例定数

比例関数の合成

φ = 比例定数 a

ψ = 比例定数 b

$\psi \circ \varphi$ 比例定数 ba

合成関数の微分

$f : \mathbb{R} \to \mathbb{R}$

$g : \mathbb{R} \to \mathbb{R}$

$(g \circ f)'(x) = g'(f(x)) f'(x)$

$g'(f(x)) \circ f'(x)$

$f'(x)$ = 比例定数

多変数

$f : \mathbb{R}^n \to \mathbb{R}^m$ $x \in \mathbb{R}^n$ で微分

$f'(x)$ $\mathbb{R}^n \to \mathbb{R}^m$ linear(行列)

$m \times n$ の行列

$$\begin{pmatrix} x^1 \\ \vdots \\ x^n \end{pmatrix} \longmapsto \begin{pmatrix} y^1(x^1 \cdots x^n) \\ \vdots \\ y^n(x^1 \cdots x^n) \end{pmatrix}$$

$$\begin{pmatrix} \frac{\partial y^1}{\partial x^1} & \frac{\partial y^1}{\partial x^2} & \frac{\partial y^1}{\partial x^3} & \cdots & \frac{\partial y^1}{\partial x^n} \\ \vdots & & & & \\ \frac{\partial y^m}{\partial x^1} & \frac{\partial y^m}{\partial x^2} & \frac{\partial y^m}{\partial x^3} & \cdots & \frac{\partial y^m}{\partial x^n} \end{pmatrix}$$

合成関数

$g : \mathbb{R}^m \to \mathbb{R}^l$ $y \in \mathbb{R}^m$ で微分

$g'(y)$ $\mathbb{R}^m \to \mathbb{R}^l$ linear

$$\begin{pmatrix} y^1 \\ \vdots \\ y^m \end{pmatrix} \longmapsto \begin{pmatrix} z^1(y^1, \cdots, y^m) \\ z^l(y^1, \cdots, y^m) \end{pmatrix}$$

$$\begin{pmatrix} \frac{\partial z^1}{\partial y^1} & \cdots & \frac{\partial z^1}{\partial y^m} \\ \frac{\partial z^l}{\partial y^1} & \cdots & \frac{\partial z^l}{\partial y^m} \end{pmatrix}$$

$g \circ f : \mathbb{R}^n \to \mathbb{R}^l$

$(g \circ f)'(x)$

$= g'(f(x)) \circ f'(x)$

$z = f(x, y)$ $f : \mathbb{R}^2 \to \mathbb{R}$

$x = \varphi(t)$

$y = \psi(t)$ $t \longmapsto \begin{pmatrix} \varphi(t) \\ \psi(t) \end{pmatrix}$ $\mathbb{R} \to \mathbb{R}^2$

$z = f(\varphi(t), \psi(t))$ $\mathbb{R} \to \mathbb{R}$

2×1の行列

$$\begin{pmatrix} \varphi'(t) \\ \psi'(t) \end{pmatrix}$$

2×1の行列

$$\left\{ \frac{\partial f}{\partial x}(\varphi(t), \psi(t)), \frac{\partial f}{\partial y}(\varphi(t), \psi(t)) \right\} \begin{pmatrix} \varphi'(t) \\ \psi'(t) \end{pmatrix}$$

$$= \frac{\partial f}{\partial x}(\varphi(t), \psi(t)) \varphi'(t) + \frac{\partial f}{\partial y}(\varphi(t), \psi(t)) \psi'(t)$$

$z = f(x, y)$

$x = \varphi(u, v)$

$y = \psi(u, v)$

$$\begin{pmatrix} u \\ v \end{pmatrix} \longmapsto \begin{pmatrix} \varphi(u, v) \\ \psi(u, v) \end{pmatrix} \quad \mathbb{R}^2 \to \mathbb{R}^2$$

$z = f(\varphi(u, v), \psi(u, v))$

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$$\left(\frac{\partial f}{\partial x}(\varphi(u,v),\psi(u,v)) \quad \frac{\partial f}{\partial y}(\varphi(u,v),\psi(u,v))\right)\begin{pmatrix}\frac{\partial \varphi}{\partial u}(u,v) & \frac{\partial \varphi}{\partial v}(u,v)\\ \frac{\partial \psi}{\partial u}(u,v) & \frac{\partial \psi}{\partial v}(u,v)\end{pmatrix}$$

$$=\left(\frac{\partial f}{\partial x}(\varphi(u,v),\psi(u,v))\frac{\partial \varphi}{\partial u}(u,v)+\frac{\partial f}{\partial y}(\varphi(u,v),\psi(u,v))\frac{\partial \psi}{\partial u}(u,v)\right) \cdots$$

Kock－Lawvereの公理

$D=\{d\in\mathbb{R} \mid d^2=0\}$ 無限小

$\varphi: D\to\mathbb{R}\ (\exists! a\in\mathbb{R})$

$\varphi(d)=\varphi(0)+ad \quad (\forall d\in D)$

$f: \mathbb{R}\to\mathbb{R}$ xで微分

$\varphi(d)=f(x+d) \qquad d\in D$

$\varphi(0)=f(x)$

$\varphi(d)=f(0)+ad \leftarrow ?$

$f(x+d)=f(x)+ad.$

$\varphi: D\to\mathbb{R}^n$

$$\varphi(d)=\begin{pmatrix}\varphi_1(d)\\ \vdots\\ \varphi_n(d)\end{pmatrix}=\begin{pmatrix}\varphi_1(0)+a_1d\\ \varphi_2(0)+a_2d\\ \vdots\\ \varphi_n(0)+a_nd\end{pmatrix}=\begin{pmatrix}\varphi_1(0)\\ \vdots\\ \varphi_n(0)\end{pmatrix}+\begin{pmatrix}a_1\\ \vdots\\ a_n\end{pmatrix}d$$

$$(\exists! a\in\mathbb{R}^n)(\forall \underset{D}{\underset{\cap}{d}})(\varphi(d)=\varphi(0)+ad)$$