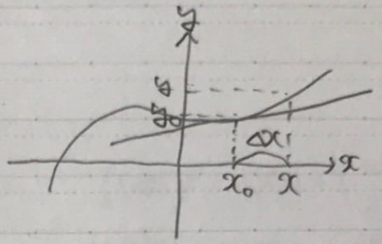
Date 5/31

第六回

微分

まがっているものをまっすぐなもので置き換える



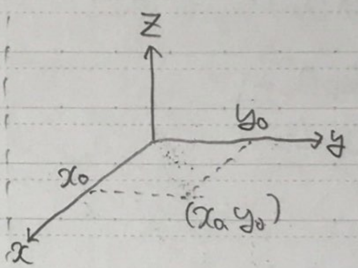
$\Delta y = y - y_0$
$= f(x) - f(x_0)$
$= g(\Delta x)$

$f'(x_0)$

$\Delta y = a\Delta x$ (a：定数)

比例定数、微分係数

$z = f(x, y)$



$\Delta z = g(\Delta x, \Delta y)$

$\Delta z = a\Delta x + b\Delta y$　　$\mathbb{R}^2 \to \mathbb{R}$ 線形写像 (mapping)

偏微分

$a = \frac{\partial f}{\partial x}(x_0, y_0)$

$b = \frac{\partial f}{\partial y}(x_0, y_0)$

1×2の行列

$f'(x_0, y_0) = \left[\frac{\partial f}{\partial x}(x_0, y_0)\ \ \frac{\partial f}{\partial y}(x_0, y_0)\right]$

線形写像　行列で表わされる

$\mathbb{R}^n \to \mathbb{R}^m$　$m \times n$の行列

(1,1)成分　(1,2)成分

足し算
スカラー倍

$\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} + \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} = \begin{pmatrix} 6 & 9 \\ 9 & 12 \end{pmatrix}$

(2,1)成分　(2,2)成分

$5\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 5 & 15 \\ 10 & 20 \end{pmatrix}$

$u = f(x, y, z)$

$\Delta u = a\Delta x + b\Delta y + c\Delta z$

(a, b, c)

$\frac{\partial f}{\partial x}(x_0, y_0, z_0)$

$(\varphi_1 + \varphi_2)(x) = \varphi_1(x) + \varphi_2(x)$

φ_1, φ_2が線形 $\Rightarrow$ $\varphi_1 + \varphi_2$が線形

→ report I

$(\alpha\varphi)(x) = \alpha\varphi(x)$

φが線形 $\Rightarrow$ $\alpha\varphi$が線形

→ report II

$\varphi_1 \Leftrightarrow \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$　　$\varphi_2 \Leftrightarrow \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$

$[(\varphi_1+\varphi_2)(e_1)\ \ (\varphi_1+\varphi_2)(e_2)]$

$= [\varphi_1(e_1)+\varphi_2(e_1)\ \ \varphi_1(e_2)+\varphi_2(e_2)]$

$= \left[\begin{pmatrix} a_{11} \\ a_{21} \end{pmatrix} + \begin{pmatrix} b_{11} \\ b_{21} \end{pmatrix}\ \ \begin{pmatrix} a_{12} \\ a_{22} \end{pmatrix} + \begin{pmatrix} b_{12} \\ b_{22} \end{pmatrix}\right]$

$= \begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{bmatrix}$

$\alpha\varphi \Leftrightarrow \begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$

各項にα倍されていることを示せ

→ report III

$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$

$= \begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} \\ a_{21}b_{11} + a_{22}b_{21} \end{pmatrix}$

$\varphi : \mathbb{R}^2 - \mathbb{R}^2$ linear
$\psi : \mathbb{R}^2 \to \mathbb{R}^2$ linear

線形：linear
たし算をかける
かけ算を前に

合成
$\psi \circ \varphi$

$\varphi \leftrightarrow A = \begin{pmatrix} a_{11}, & a_{12} \\ a_{21}, & a_{22} \end{pmatrix} = (\varphi(e_1), \varphi(e_2))$

$\psi \leftrightarrow B = \begin{pmatrix} b_{11} \cdot & b_{12} \\ b_{21}, & b_{22} \end{pmatrix} = (\psi(e_1), \psi(e_2))$

$(\psi \circ \varphi)(e_1)$
$(\psi \circ \varphi)(e_2)$

$((\psi \circ \varphi)(e_1), (\psi \circ \varphi)(e_2))$

$$(\psi \circ \varphi)(e_1) = \psi(\varphi(e_1)) = \begin{pmatrix} b_{11}, & b_{12} \\ b_{21} \cdot & b_{22} \end{pmatrix}\begin{pmatrix} a_{11} \\ a_{21} \end{pmatrix} = \begin{pmatrix} b_{11}a_{11} + b_{12}a_{21} \\ b_{21}a_{11} + b_{22}a_{21} \end{pmatrix}$$

$$(\psi \circ \varphi)(e_2) = \psi(\varphi(e_2)) = \begin{pmatrix} b_{11}, & b_{12} \\ b_{21}, & b_{22} \end{pmatrix}\begin{pmatrix} a_{12} \\ a_{22} \end{pmatrix} = \begin{pmatrix} b_{11}a_{12} + b_{12}a_{22} \\ b_{21}a_{12} + b_{22}a_{22} \end{pmatrix}$$

$$\begin{pmatrix} \boxed{\begin{matrix} b_{11}a_{11} + b_{21}a_{21}, \\ b_{21}a_{11} + b_{22}a_{21}, \end{matrix}} & \boxed{\begin{matrix} b_{11}a_{12} + b_{12}a_{22} \\ b_{21}a_{12} + b_{22}a_{22} \end{matrix}} \end{pmatrix}$$

合成関数の微分

$f_i : \mathbb{R}^n \to \mathbb{R}$

$f(x_1, \cdots, x_n)$ $\quad f : \mathbb{R}^n \to \mathbb{R}^m$ $\quad f = \begin{pmatrix} f_1 \\ \vdots \\ f_n \end{pmatrix}$

$f'(x_1^0, \cdots, x_n^0)$ $\quad m \times n$の行列

$$= \begin{bmatrix} \frac{\partial f_1}{\partial x_1}(x_1^0, \cdots, x_n^0) & \cdots\cdots & \frac{\partial f_1}{\partial x_n}(x_1^0, \cdots, x_n^0) \\ \frac{\partial f_n}{\partial x_1}(x_1^0, \cdots, x_n^0) & \cdots\cdots & \frac{\partial f_n}{\partial x_n}(x_1^0, \cdots, x_n^0) \end{bmatrix}$$

ex) IV (1)

$z = xy^2 - x^2y$ $\quad \mathbb{R}^2 \to \mathbb{R}$

$x = t^2,\ y = e^t$ ➡ $t \to \begin{pmatrix} x \\ y \end{pmatrix} = \mathbb{R} \to \mathbb{R}^2$

$z = f(x, y)$
$x = \varphi(t)$
$y = \psi(t)$

$\frac{\partial f}{\partial x} x' + \frac{\partial f}{\partial y} y'$

$\frac{\partial z}{\partial x} = y^2 - 2xy$ $\qquad x' = \frac{dx}{dt}$

ex) V (1)

$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial u}$

$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial v} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial v}$

$g \circ f$ $\quad f : \mathbb{R} \to \mathbb{R}$
$\quad g : \mathbb{R} \to \mathbb{R}$

$(g \circ f)'(x) = g'(f(x))f'(x)$

$f : \mathbb{R}^n \to \mathbb{R}^m$
$g : \mathbb{R}^m \to \mathbb{R}^l$
$g \circ f : \mathbb{R}^n \to \mathbb{R}^l$
$x \in \mathbb{R}^n$で微分

$f'(x) : \mathbb{R}^n \to \mathbb{R}^m$ linear $\quad g'(y) : \mathbb{R}^m \to \mathbb{R}^l$ linear
$(g \circ f)'(x) = g'(f(x))(f'(x))$

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