Date

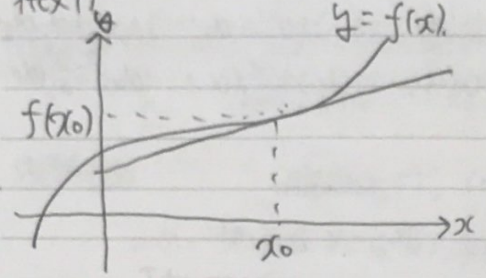
線形代数：まっすぐなものの代数

多変数の微分

$Z = f(x, y)$　2変数

$f(x, a)$　(aは定数)

微分



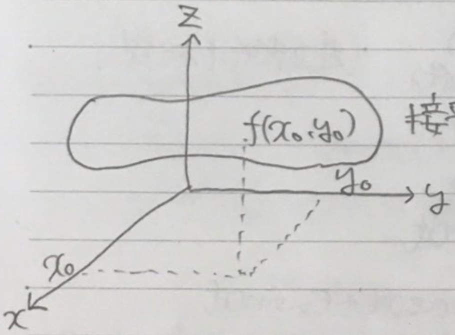
曲がっている

→曲がっているのは嫌なのでまっすぐになるよう接線でおきかえる

$y - f(x_0) = a(x - x_0)$

↑ 微分係数 $f'(x_0)$

2変数



接平面

$z - f(x_0, y_0) = a(x - x_0) + b(y - y_0)$

aとbは定数

~~y~~ x 切りする ($y = y_0$ でみる、固定)

$x \longrightarrow f(x, x_0)$　一変数にする

x方向の偏微分

$$a = \frac{\partial f}{\partial x}(x_0, y_0)$$

y方向の偏微分

$$b = \frac{\partial f}{\partial y}(x_0, y_0)$$

3変数(グラフは書けない)

比例だと…

$y = ax$ ($y = f(x)$)

$f(x_1 + x_2) = f(x_1) + f(x_2)$

$f(\alpha x) = \alpha f(x)$　$\mathbb{R} \to \mathbb{R}$　線形関数

$$\underbrace{(y - y_0)}_{\Delta y} = g\underbrace{(x - x_0)}_{\Delta x}$$

$\Delta y = a \Delta x$ (a: 比例定数)　2次元から1次元 $\mathbb{R}^2 \to \mathbb{R}$

$\varphi(x_1 + x_2) = \varphi(x_1) + \varphi(x_2)$ …①

$\varphi(\alpha x) = \alpha \varphi(x)$ (α: 実数) …②

$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = x_1 e_1 + x_2 e_2$$

$$\begin{aligned} \varphi(x) &= \varphi(x_1 e_1 + x_2 e_2) \\ &= \varphi(x_1 e_1) + \varphi(x_2 e_2) \quad (\because ①) \\ &= x_1 \underbrace{\varphi(e_1)}_{\mathbb{R}^2} + x_2 \underbrace{\varphi(e_2)}_{\mathbb{R}^2} \quad (\because ②) \\ &= x_1 a + x_2 b \end{aligned}$$

$\varphi(e_1) = a$, $\varphi(e_2) = b$

$$\therefore [a, b] \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = a x_1 + b x_2$$

1×2の行列

$\varphi: \mathbb{R}^2 \to \mathbb{R}^2$

$$\varphi(e_1) = \begin{pmatrix} a_{11} \\ a_{21} \end{pmatrix} \qquad \varphi(e_2) = \begin{pmatrix} a_{12} \\ a_{22} \end{pmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} a_{11} x_1 + a_{12} x_2 \\ a_{21} x_1 + a_{22} x_2 \end{pmatrix}$$

2×2の行列

→線形だと2×2の行列で表すことができる

$Z_0 = f(x_0, y_0)$

2変数

$$\underbrace{Z - Z_0}_{\Delta Z} = a \underbrace{x - x_0}_{\Delta x} + b \underbrace{y - y_0}_{\Delta y}$$

$$\Delta Z = [a, b] \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}$$

$u=f(x,y,z)$

$\Delta u = a\Delta x + b\Delta y + c\Delta z$

(a, b, c)

$\frac{\partial f}{\partial x}(x_0, y_0, z_0)$

$(\varphi_1+\varphi_2)(x) = \varphi_1(x)+\varphi_2(x)$

φ_1, φ_2が線形 $\Rightarrow \varphi_1+\varphi_2$が線形

→ report I

$(\alpha\varphi)(x) = \alpha\varphi(x)$

φが線形 $\Rightarrow \alpha\varphi$が線形

→ report II

$\varphi_1 \Leftrightarrow \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ $\varphi_2 \Leftrightarrow \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$

$$[(\varphi_1+\varphi_2)(e_1) \quad (\varphi_1+\varphi_2)(e_2)]$$
$$= [\varphi_1(e_1)+\varphi_2(e_1) \quad \varphi_1(e_2)+\varphi_2(e_2)]$$
$$= \left[\begin{pmatrix} a_{11} \\ a_{21} \end{pmatrix}+\begin{pmatrix} b_{11} \\ b_{21} \end{pmatrix} \quad \begin{pmatrix} a_{12} \\ a_{22} \end{pmatrix}+\begin{pmatrix} b_{12} \\ b_{22} \end{pmatrix}\right]$$
$$= \begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{bmatrix}$$

$\alpha\varphi \Leftrightarrow \begin{pmatrix} \ & \ \\ \ & \ \end{pmatrix}$

各項にα倍されていることを示せ

→ report III

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$$
$$= \begin{pmatrix} a_{11}b_{11}+a_{12}b_{21} \\ a_{21}b_{11}+a_{22}b_{21} \end{pmatrix}$$