5/10

$d_1 \in D$

$f(x_0+d_1) = f(x_0) + f'(x_0)d_1$

$d_1, d_2 \in D$

$f(x_0+d_1+d_2) = f(x_0)+f'(x_0)(d_1+d_2)+\boxed{f''(x_0)d_1d_2}$

$= f(x_0)+f'(x_0)(d_1+d_2)+\boxed{\frac{1}{2}f''(x_0)(d_1+d_2)^2}$

$d_1d_2 = \frac{(d_1+d_2)^2}{2}$

$f(x_0+d_1+d_2+d_3) = f(x_0)+f'(x_0)(d_1+d_2+d_3)$

$+f''(x_0)\boxed{d_1d_2+d_1d_3+d_2d_3}+f'''(x_0)\boxed{d_1d_2d_3}$

$(d_1+d_2+d_3)^2 = \underbrace{d_1^2}_{0}+\underbrace{d_2^2}_{0}+\underbrace{d_3^2}_{0}+2(d_1d_2+d_2d_3+d_3d_1)$

$\Rightarrow d_1d_2+d_2d_3+d_3d_1 = \frac{1}{2}(d_1+d_2+d_3)^2$

$(d_1+d_2+d_3)^3 = \underbrace{d_1^3}_{0}+\underbrace{d_2^3}_{0}+\underbrace{d_3^3}_{0}+3(\underbrace{d_1^2d_2}_{0}+\underbrace{d_2^2d_3}_{0}+\underbrace{d_3^2d_1}_{0})+6d_1d_2d_3$

$d_1d_2d_3 = \frac{1}{6}(d_1+d_2+d_3)^3$

$f(x_0+d_1+d_2+d_3+d_4)$

$= f(x_0)+f'(x_0)(d_1+d_2+d_3+d_4)+\frac{1}{2}f''(x_0)(d_1+d_2+d_3+d_4)^2$

$+\frac{1}{3!}f'''(x_0)(d_1+d_2+d_3+d_4)^3+\frac{1}{4!}f''''(x_0)(d_1+d_2+d_3+d_4)^4$

→ report I

$f(x_0+d_1+d_2+\cdots+d_n)$

$= f(x_0)+f'(x_0)(d_1+\cdots+d_n)+\frac{1}{2}f''(x_0)(d_1+\cdots+d_n)^2$

$+\cdots+\frac{1}{i!}f^{(i)}(x_0)+(d_1+\cdots+d_n)^i+\cdots+\frac{1}{n!}f^{(n)}(x_0)(d_1+\cdots+d_n)^n$

Taylor展開

無限小のLevelで多項式に展開

n次の多項式と仮定

$f(x) = a_nx^n+a_{n-1}x^{n-1}+\cdots+a_1x+a_0$

$a_0 = f(0)$

$f'(x) = na_nx^{n-1}+\cdots+a_1$

$a_1 = f'(0)$

$f''(x) = n(n-1)a_nx^{n-2}+\cdots+2a_2$

$a_2 = \frac{f''(0)}{2}$

$a_3 = \frac{f'''(0)}{3!}$

一般的に、$a_i = \frac{f^{(i)}(0)}{i!}$

無限次の多項式

$f(x) = e^x = a_0+a_1x+a_2x^2+\cdots$

$1 = e^0 = f(0) = a_0$

$a_1 = f'(0) = 1 \quad a_2 = \frac{1}{2}f''(0) = \frac{1}{2} \quad a_3 = \frac{1}{3!}f'''(0) = \frac{1}{3!}$

$a_n = \frac{1}{n!}$

$\rightarrow e^x = 1+x+\frac{1}{2}x^2+\frac{1}{3!}x^3+\cdots+\frac{1}{n!}x^n+\cdots$

$\cos x = b_0+b_1x+b_2x^2+\cdots$

$= 1-\frac{1}{2}x^2+\frac{1}{4!}x^4-\frac{1}{6!}x^6+\cdots$

$\sin x = c_0+c_1x+c_2x^2+\cdots$

$= x-\frac{1}{3!}x^3+\frac{1}{5!}x^5-\frac{1}{7!}x^7+\cdots$

→ report II

$e^x \rightarrow e^z$ (実数→複素数)

θ:実数

$e^{i\theta} = 1+i\theta-\frac{1}{2}\theta^2-\frac{1}{3!}i\theta^3+\frac{1}{4!}\theta^4+\frac{1}{5!}\theta^5 i+\cdots$

$= \underbrace{\boxed{1-\frac{1}{2}\theta^2+\frac{1}{4!}\theta^4-\frac{1}{6!}\theta^6+\cdots}}_{\cos\theta}+\underbrace{\boxed{\theta-\frac{1}{3!}\theta^3+\frac{1}{5!}\theta^5+\cdots}}_{\sin\theta}i$

$= \cos\theta + i\sin\theta$

Euler(オイラー)

指数法則

$e^{z_1+z_2} = e^{z_1}e^{z_2}$

$e^{i(\theta_1+\theta_2)} = e^{i\theta_1+i\theta_2} = e^{i\theta_1}e^{i\theta_2}$

$\cos(\theta_1+\theta_2)+i\sin(\theta_1+\theta_2)$

$(\cos\theta_1+i\sin\theta_1)(\cos\theta_2+i\sin\theta_2)$

$=(\cos\theta_1\cos\theta_2-\sin\theta_1\sin\theta_2)$

$+i(\sin\theta_1\cos\theta_2+\cos\theta_1\sin\theta_2)$

加法定理

指数法則の証明

$e^{z_1} = 1+z_1+\frac{1}{2}z_1^2+\frac{1}{3!}z_1^3+\cdots$

$e^{z_2} = 1+z_2+\frac{1}{2}z_2^2+\frac{1}{3!}z_2^3+\cdots$

$e^{z_1+z_2} = 1+(z_1+z_2)+\frac{1}{2}(z_1+z_2)^2+\cdots+\frac{1}{n!}(z_1+z_2)^n$

z_1とz_2の多項式

係数が同じであればよい

$z_1^i \cdot z_2^j \quad (i+j=1)$

$\frac{1}{i!} \quad \frac{1}{j!}$ (右辺)

$\frac{1}{n!}(z_1+z_2)^n$

→二項定理で展開

→ report III