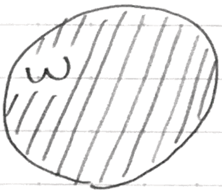
第24回 1/31

2/8 自然B211 にてレポート提出

γ：閉曲線

f：解析

$\int_\gamma f(z)dz=0$



$\int_\gamma \frac{f(z)}{z-w}dz$

$=2\pi i f(w)$

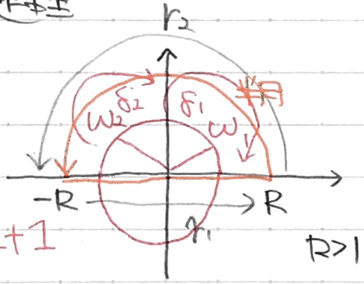
$\therefore f(w)=\frac{1}{2\pi i}\int_\gamma \frac{f(z)}{z-w}dz$

多項式，代数の基本定理

線積分

$$\boxed{\int_\gamma \frac{z^2}{1+z^4}dz=\frac{\pi}{\sqrt{2}}}$$

report 1



$\int_{-\infty}^{\infty}\frac{x^2}{1+x^2}dx=\frac{\pi}{\sqrt{2}}$

$\int_\gamma \frac{z^2}{1+z^4}dz=\underbrace{\int_{\gamma_1}\frac{z^2}{1+z^4}dz}_{\shortparallel}+\int_{\gamma_2}\frac{z^2}{1+z^4}dz$

$\int_{-R}^{R}\frac{x^2}{1+x^4}dz=\int_{-\infty}^{\infty}\frac{x^2}{1+x^4}dz$

$\int_{\gamma_2}\frac{z^2}{1+z^4}dz$

半径

$\left|\frac{z^2}{1+z^4}\right|=\frac{|z|^2}{|1+z^4|}\leq\frac{|R|^2}{|z^4+1|}\cdot\pi R$

$z^4=(z^4+1)-1$

$R^4=|z^4|\leq|z^4+1|+1$

$R^4-1\leq|z^4+1|$

← 三角不等式

$|f(x)|\leq M$

$\left|\int_a^b f(x)dx\right|\leq M(b-a)$

$\int_\gamma f(z)dz\leq M\times(\gamma$の長さ$)$

$\therefore \int_{\gamma_2}\frac{z^2}{1+z^4}dz\leq\frac{R^2}{R^4-1}\cdot\pi R \quad \cdots(*)$

$R\to\infty$ すると $(*)\to 0$

ゆえに $\int_\gamma \frac{z^2}{1+z^4}dz=\int_{-\infty}^{\infty}\frac{x^2}{1+x^4}dz$

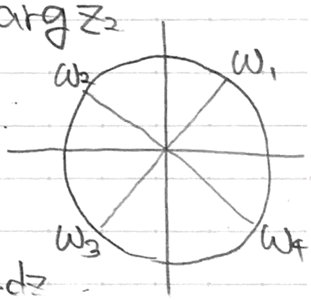
($R\to\infty$にすると近似出来る)

$1+z^4=0$

$z^2=\pm i$

$|z_1z_2|=|z_1||z_2|$

$\arg(z_1z_2)=\arg z_1+\arg z_2$



$\int_{\gamma\cup\delta_1\cup\delta_2}\frac{z^2}{1+z^4}dz=0$

$=\int_\gamma\frac{z^2}{1+z^4}dz-\int_{\delta_1}\frac{z^2}{1+z^4}dz$

$-\int_{\delta_2}\frac{z^2}{1+z^4}dz=0$

$\arg i=\frac{\pi}{2}+2\pi$

$=\frac{5}{2}\pi$

$\int_\gamma\frac{z^2}{1+z^4}dz=\int_{\delta_1}\frac{z^2}{1+z^4}dz+\int_{\delta_2}\frac{z^2}{1+z^4}dz$

$\frac{z^2}{1+z^4}=\frac{z^2}{(z-w_1)(z-w_2)(z-w_3)(z-w_4)}$

$=\frac{1}{z-w_1}\underbrace{\frac{1}{(z-w_2)(z-w_3)(z-w_4)}}_{g(z)}$

$\int_{\delta_1}\frac{g(z)}{z-w_1}dz=g(w_1)\cdot 2\pi i$

$w_1=\frac{1+i}{\sqrt{2}}$

$\frac{z^2}{1+z^4}=\frac{1}{z-w_2}\underbrace{\frac{1}{(z-w_1)(z-w_3)(z-w_4)}}_{h(z)}$

$\int_{\delta_2}\frac{h(z)}{z-w_2}=h(w_2)\cdot 2\pi i$

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$$\int_0^\infty \frac{x^2}{x^4+x^2+1}\,dx \text{ を求めよ}$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} \frac{x^2}{x^4+x^2+1}\,dx$$