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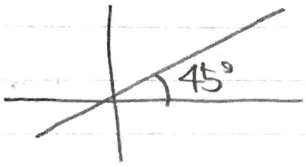
$$\varphi(a,b)=\varphi(a_1e_1+a_2e_2,\ b_1e_1+b_2e_2)$$
$$=a_1b_1\varphi(e_1,e_1)+a_1b_2\varphi(e_1,e_2)$$
$$+a_2b_1\varphi(e_2,e_1)+a_2b_2\varphi(e_2,e_2)$$
$$=(a_1b_2-a_2b_1)\varphi(e_1,e_2) \quad >0$$

$$\left(\begin{pmatrix}a_1\\a_2\end{pmatrix},\begin{pmatrix}b_1\\b_2\end{pmatrix}\right)\longmapsto a_1b_2-a_2b_1=\begin{vmatrix}a_1 & b_1\\ a_2 & b_2\end{vmatrix}$$

複素数の微積分

$f:\mathbb{R}\to\mathbb{R}$

$df=f'dx$



線型写像 $\mathbb{R}\to\mathbb{R}$

$df(x)=f'(x)dx$

微分係数

$f:\mathbb{C}\to\mathbb{C}$, $f=f_1+if_2$ (実部, 虚部)

$z=x+iy$　$f_1:\mathbb{C}\to\mathbb{R}$　$f_2:\mathbb{C}\to\mathbb{R}$

$$df=df_1+idf_2$$
$$=\frac{\partial f_1}{\partial x}dx+\frac{\partial f_1}{\partial y}dy+i\left\{\frac{\partial f_2}{\partial x}dx+\frac{\partial f_2}{\partial y}dy\right\}$$

$dz=dx+idy$

$dx: x+iy\longmapsto x$

$dy: x+iy\longmapsto y$

$dz(x+iy)=x+iy$

$(dx+idy)(x+iy)=x+iy$

$$=g\,dz \qquad g:\mathbb{C}\to\mathbb{C}$$
$$=(g_1+ig_2)(dx+idy)$$
$$=(g_1dx-g_2dy)+i(g_2dx+g_1dy)$$

$$\Rightarrow \frac{\partial f_1}{\partial x}=\frac{\partial f_2}{\partial y},\quad \frac{\partial f_2}{\partial x}=-\frac{\partial f_1}{\partial y}.$$

Cauchy-Riemannの方程式　正則 / 解析 } 関数

$\Rightarrow \dfrac{\partial f_1}{\partial x}-i\dfrac{\partial f_1}{\partial y}$　微分係数 比例定数

正則関数の例)，定値関数

$$f(z)=C=C_1+iC_2$$
$$f'(z)=0$$

$f(z)=z=x+iy$　$\left(\begin{matrix}f_1(z)=x\\ f_2(z)=y\end{matrix}\right)$

$\dfrac{\partial f_1}{\partial x}=1$, $\dfrac{\partial f_1}{\partial y}=0$.

$\dfrac{\partial f_2}{\partial x}=0$, $\dfrac{\partial f_2}{\partial y}=1$.

$f'(z)=1$　$(1-i\cdot 0=1)$

f, gも正則 $\Rightarrow$ $f+g$も正則

$$d(f+g)=df+dg$$
$$=f'dz+g'dz$$
$$=(f'+g')dz$$

$(f+g)'=f'+g'$

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f: 正則 α: 複素数 $\Rightarrow$ αf 正則

$(\alpha f)'=\alpha f'$

$$df=\frac{\partial f}{\partial x}dx+\frac{\partial f}{\partial y}dy$$

$d(fg)$

$$d(fg)=\frac{\partial(fg)}{\partial x}dx+\frac{\partial(fg)}{\partial y}dy$$
$$=\left(\frac{\partial f}{\partial x}g+\frac{\partial g}{\partial x}f\right)dx+\left(\frac{\partial f}{\partial y}g+\frac{\partial g}{\partial y}f\right)dy$$
$$=\left(\frac{\partial f}{\partial x}dx+\frac{\partial f}{\partial y}dy\right)g+\left(\frac{\partial g}{\partial x}dx+\frac{\partial g}{\partial y}dy\right)f$$
$$=(df)g+(dg)f$$

f,g: 正則 $\to$ fg: 正則

$$d(fg)=(df)g+f\,dg$$
$$=(f'dz)g+f(g'dz)$$
$$=(f'g+fg')dz$$

$(fg)'=f'g+fg'$

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$$d\left(\frac{f}{g}\right)=\frac{\partial}{\partial x}\left(\frac{f}{g}\right)dx+\frac{\partial}{\partial y}\left(\frac{f}{g}\right)dy$$

$$=\frac{\frac{\partial f}{\partial x}g-f\frac{\partial g}{\partial x}}{g^2}dx+\frac{\frac{\partial f}{\partial y}g-f\frac{\partial g}{\partial y}}{g^2}dy$$

$$=\frac{1}{g^2}\left\{\left(\frac{\partial f}{\partial x}dx+\frac{\partial f}{\partial y}dy\right)g-f\left(\frac{\partial g}{\partial x}dx+\frac{\partial g}{\partial y}dy\right)\right\}$$

$$=\frac{1}{g^2}\{(df)g-fdg\}$$

f, g : 正則, $g\neq 0$ $\Rightarrow$ $\frac{f}{g}$: 正則

$$=\frac{1}{g^2}\{(f'dz)g-fg'dz\}$$

$$=\frac{f'g-fg'}{g^2}$$

$$f(z)=a_nz^n+a_{n-1}z^{n-1}+\cdots+a_0$$

$$f'(z)=na_nz^{n-1}+(n-1)a_{n-1}z^{n-2}+\cdots+az$$

無限次の多項式

$$\sin x=a_0+a_1x+a_2x^2+\cdots$$

$$\cos x=b_0+b_1x+b_2x^2+\cdots$$

$$e^x=1+x+\frac{1}{2!}x^2+\frac{1}{3!}x^3+\cdots$$

θは実数

$$e^{i\theta}=1+i\theta+\frac{1}{2}(i\theta)^2+\frac{1}{3!}(i\theta)^3+\frac{1}{4!}(i\theta)^4$$

$$=1+i\theta-\frac{1}{2}\theta^2-\frac{i}{3!}\theta^3+\frac{1}{4!}\theta^4$$

$$=\underbrace{\left(1-\frac{1}{2}\theta^2+\frac{1}{4!}\theta^4\right)}_{\cos\theta}+i\underbrace{\left(\theta-\frac{1}{3!}\theta^3\right)}_{\sin\theta}$$

$$=\cos\theta+i\sin\theta$$

$$e^{i(\theta_1+\theta_2)}=e^{i\theta_1}e^{i\theta_2}$$

$$\cos(\theta_1+\theta_2)+i\sin(\theta_1+\theta_2)=(\cos\theta_1+i\sin\theta_1)(\cos\theta_2+i\sin\theta_2)$$

→ 加法定理

$$e^{z_1+z_2}=e^{z_1}e^{z_2}$$

$$=\left(1+z_1+\frac{1}{2!}z_1^2+\cdots\right)\left(1+z_2+\frac{1}{2!}z_2^2+\cdots\right)$$

$z_1^k \cdot z_2^l$ $\quad \frac{1}{k!}\cdot\frac{1}{l!}$

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$$1+(z_1+z_2)+\frac{1}{2}(z_1+z_2)^2+\frac{1}{3!}(z_1+z_2)^3\cdots$$

係数を比べる

$(z_1+z_2)^n$

$k+l=n$ 2項定理

$z_1^kz_2^l$

$$\frac{1}{n!}(z_1+z_2)^n$$