18〜

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第20回 12/6

ベクトル解析

空間 $\mathbb{R}^3$

ベクトル $e_1=\begin{pmatrix}1\\0\\0\end{pmatrix}, e_2=\begin{pmatrix}0\\1\\0\end{pmatrix}, e_3=\begin{pmatrix}0\\0\\1\end{pmatrix}$

$a=a_1e_1+a_2e_2+a_3e_3$

1対1の対応
同一視

一次の交代形式

$\mathbb{R}^3\to\mathbb{R}$ 線型写像

$a_1, a_2, a_3 \in \mathbb{R}$

$\omega=a_1dx+a_2dy+a_3dz$

2次の交代形式 $\mathbb{R}^3\times\mathbb{R}^3\to\mathbb{R}$

$a_1dy\wedge dz+a_2dz\wedge dx+a_3dx\wedge dy$

$a=a_1$：基底

3次の交代形式 $\mathbb{R}^3\times\mathbb{R}^3\times\mathbb{R}^3\to\mathbb{R}$

$a\,dx\wedge dy\wedge dz$ ← 1対1の対応

(スカラー場)

grad

0次の微分形式 $\xrightarrow{d}$

(ベクトル場) $f_i:\mathbb{R}^3\to\mathbb{R}$ $\xrightarrow{rot}$ (ベクトル場) $\xrightarrow{div}$

$(a_1e_1+a_2e_2+a_3e_3)$　　$f_1e_1+f_2e_2+f_3e_3$

$f_1e_1+f_2e_2+f_3e_3$　　↕

1次の微分形式 $\xrightarrow{d}$ 2次の微分形式 $\xrightarrow{d}$

$f_1dx+f_2dy+f_3dz$　　$f_1dy\wedge dz+f_2dz\wedge dx+f_3dx\wedge dy$

→(スカラー場)

f

↕

→3次の微分形式

$f\,dx\wedge dy\wedge dz$

$\varphi\wedge\psi=-\psi\wedge\varphi$

$d\varphi=\frac{\partial\varphi}{\partial x}dx+\frac{\partial\varphi}{\partial y}dy+\frac{\partial\varphi}{\partial z}dz$

$d(f_1dx+f_2dy+f_3dz)$

$=(df_1)\wedge dx+(df_2)\wedge dy+(df_3)\wedge dz$

$=\underset{\overset{\shortparallel}{0}}{\frac{\partial f_1}{\partial x}dx\wedge dx}+\frac{\partial f_1}{\partial y}\underset{-dx\wedge dy}{\underline{dy\wedge dx}}+\frac{\partial f_1}{\partial z}dz\wedge dx$

$+\frac{\partial f_2}{\partial x}dx\wedge dy+\frac{\partial f_2}{\partial y}\underset{\overset{\shortparallel}{0}}{dy\wedge dy}+\frac{\partial f_2}{\partial z}\underset{-dy\wedge dz}{\underline{dz\wedge dy}}$

$+\frac{\partial f_3}{\partial x}\underset{-dz\wedge dx}{\underline{dx\wedge dz}}+\frac{\partial f_3}{\partial y}dy\wedge dz+\frac{\partial f_3}{\partial z}\underset{\overset{\shortparallel}{0}}{dz\wedge dz}$

→ report I
grad の証明

$\left(\frac{\partial f_1}{\partial x}dx+\frac{\partial f_1}{\partial y}dy+\frac{\partial f_1}{\partial z}dz\right)\wedge dy\wedge dz$

$+\left(\frac{\partial f_2}{\partial x}dx+\frac{\partial f_2}{\partial y}dy+\frac{\partial f_2}{\partial z}dz\right)\wedge dz\wedge dx$

$+\left(\frac{\partial f_3}{\partial x}dx+\frac{\partial f_3}{\partial y}dy+\frac{\partial f_3}{\partial z}dz\right)\wedge dx\wedge dy$

$=\frac{\partial f_1}{\partial x}dx\wedge dy\wedge dz+\frac{\partial f_1}{\partial y}\underset{\overset{\shortparallel}{0}}{dy\wedge dy}\wedge dz+\frac{\partial f_1}{\partial z}\underset{\overset{\shortparallel}{0}}{dz\wedge dy\wedge dz}$

$+\frac{\partial f_2}{\partial y}dy\wedge dz\wedge dx+\frac{\partial f_3}{\partial z}dz\wedge dx\wedge dy$

$\left(\begin{array}{l}dy\wedge dx\wedge dz\\=-dx\wedge dy\wedge dz\end{array}\right)$　→ report II
rot の証明

複素数の微積分

$z\in\mathbb{C}$, $x,y\in\mathbb{R}$.

$z=x+yi$　$i^2=-1$

$\mathbb{R}^2\to\mathbb{R}$ 線型 1次の交代形式

$\mathbb{R}^2\times\mathbb{R}^2\to\mathbb{R}$ = 双線型 2次の交代形式

$\varphi(a,b)=-\varphi(b,a)$

$a=a_1e_1+a_2e_2$

$=a_1\begin{pmatrix}1\\0\end{pmatrix}+a_2\begin{pmatrix}0\\1\end{pmatrix}$

$\varphi(a)=\varphi(a_1e_1+a_2e_2)$　　$\begin{pmatrix}a_1\\a_2\end{pmatrix}\mapsto a_1$

$=a_1\underset{\alpha_1}{\varphi(e_1)}+a_2\underset{\alpha_2}{\varphi(e_2)}$

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$$\varphi(\mathbf{a}, \mathbf{b}) = \varphi(a_1 e_1 + a_2 e_2, b e_1 + b_2 e_2)$$
$$= a_1 b_1 \varphi(e_1, e_1) + a_1 b_2 \varphi(e_1, e_2)$$
$$+ a_2 b_1 \varphi(e_2, e_1) + a_2 b_2 \varphi(e_2, e_2)$$
$$= (a_1 b_2 - a_2 b_1) \varphi(e_1, e_2) \quad 0$$

$$\left(\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \right) \mapsto a_1 b_2 - a_2 b_1 = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$