Date

$$dy\wedge dz\left(\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix},\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}\right)\mapsto\begin{vmatrix}a_2&b_2\\a_3&b_3\end{vmatrix}=1$$

$$dx\wedge dy \mapsto \begin{vmatrix}a_1&b_1\\a_2&b_2\end{vmatrix}=0$$

$$dz\wedge dx \mapsto \begin{vmatrix}a_3&b_3\\a_1&b_1\end{vmatrix}=0$$

第19回 11/22

stokesの定理

閉曲線γを縁とする

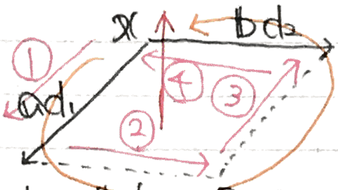
曲面σ

ベクトル場 $f:\mathbb{R}^3\to\mathbb{R}^3$

$$\int_\sigma (\mathrm{rot}\, f)\cdot dS=\int_\gamma f\, dr$$

面積分　　線積分

無限小のレベルで成り立てば



$d_1, d_2\in D$

ad_1 と bd_2 で張れる平行四辺形

$$\underset{①}{f(x)\cdot ad_1}+\underset{②}{f(x+ad_1)\cdot bd_2}$$
$$-\underset{③}{f(x+bd_2)\cdot ad_1}-\underset{④}{f(x)\cdot bd_2}$$

$$=\{\underbrace{\boxed{f(x+ad_1)-f(x)}}_{\text{微分のカタチ}}\}bd_2-\{\underbrace{\boxed{f(x+bd_2)-f(x)}}_{\text{微分のカタチ}}\}\cdot ad_1$$
$$=(f'(x)(a)d_1)\cdot bd_2-(f'(x)(b)d_2)ad_1$$
$$=(f'(x)(a)\cdot b-f'(x)(b)\cdot a)d_1d_2$$

$\varphi(a,b)$

$(a,b)\mapsto f'(x)(a)\cdot b-f'(x)(b)a$　2次の交代形式

$\in \mathbb{R}^3\times\mathbb{R}^3$

$$\varphi(a_1+a_2,b)=\varphi(a_1,b)+\varphi(a_2,b)$$
$$f'(x)(a_1+a_2)\cdot b-f'(x)(b)\cdot(a_1+a_2)$$
$$f'(x)(a_1)+f'(x)(a_2)-\{f'(x)(b)\cdot a_1+f'(x)(b)\cdot a_2\}$$
$$f'(x)(a_1)\cdot b+f'(x)(a_2)\cdot b$$
$$=\varphi(a_1,b)+\varphi(a_2,b)$$

交代

$$\varphi(b,a)=f'(x)(b)\cdot a-f'(x)(a)\cdot b=-\varphi(a,b)$$

$$\varphi=\alpha_1\underset{e_2,e_3}{dy\wedge dz}+\alpha_2\underset{e_3\ e_1}{dz\wedge dx}+\alpha_3\underset{e_1\ e_2}{dx\wedge dy}$$

$$e_2=\begin{pmatrix}0\\1\\0\end{pmatrix},\ e_3=\begin{pmatrix}0\\0\\1\end{pmatrix},\ \begin{pmatrix}0\\0\\1\end{pmatrix}$$

$$\alpha_1=\varphi(e_2,e_3)$$
$$\alpha_2=\varphi(e_3,e_1)$$
$$\alpha_3=\varphi(e_1,e_2)$$

y成分　打消

$$\varphi(e_2,e_3)=f'(x)(e_2)\cdot e_3-f'(x)(e_3)\cdot e_2$$

$$f=\begin{pmatrix}f_1\\f_2\\f_3\end{pmatrix}\qquad \frac{\partial f}{\partial y}(x)\cdot e_3=\begin{pmatrix}\frac{\partial f_1}{\partial y}(x)\\\frac{\partial f_2}{\partial y}(x)\\\frac{\partial f_3}{\partial y}(x)\end{pmatrix}\cdot e_3=\frac{\partial f_3}{\partial y}(x)$$

$$f(x+e_2d)-f(x)$$
$$f\left(\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}+\begin{pmatrix}0\\1\\0\end{pmatrix}d\right)-f\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$$
$$f\begin{pmatrix}x_1\\x_2+d\\x_3\end{pmatrix}-f\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$$

$$\frac{\partial f_2}{\partial z}(x)$$

$$\alpha_1=\frac{\partial f_3}{\partial y}(x)-\frac{\partial f_2}{\partial z}(x)$$
$$\alpha_2=\frac{\partial f_1}{\partial z}(x)-\frac{\partial f_3}{\partial x}(x)\qquad \alpha_3=\frac{\partial f_2}{\partial x}(x)-\frac{\partial f_1}{\partial y}(x)$$

report I　α_2 と α_3 を計算せよ

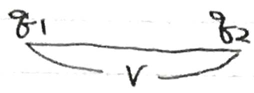
$$(\alpha_1\, dy\wedge dz+\alpha_2\, dz\wedge dx+\alpha_3\, dx\wedge dy)(a,b)d_1d_2$$
$$\alpha_1(dy\wedge dz)(a,b)$$
$$\alpha_1\begin{vmatrix}a_2d_1&b_2d_2\\a_3d_1&b_3d_2\end{vmatrix}+\alpha_2\begin{vmatrix}a_3d_1&b_3d_2\\a_1d_1&b_1d_2\end{vmatrix}+\alpha_3\begin{vmatrix}a_1d_1&b_1d_2\\a_2d_1&b_2d_2\end{vmatrix}$$

$$=\underbrace{\begin{pmatrix}\alpha_1\\\alpha_2\\\alpha_3\end{pmatrix}}_{f\text{の回転}}\begin{pmatrix}\begin{vmatrix}a_2d_1&b_2d_2\\a_3d_1&b_3d_2\end{vmatrix}\\\begin{vmatrix}a_3d_1&b_3d_2\\a_1d_1&b_1d_2\end{vmatrix}\\\begin{vmatrix}a_1d_1&b_1d_2\\a_2d_1&b_2d_2\end{vmatrix}\end{pmatrix}$$

report II　ガウスの発散定理の証明

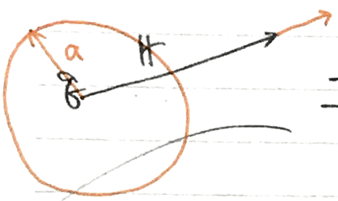
ガウスの発散定理

・クーロンの法則　　　　・万有引力の法則



$\frac{k q_1 q_2}{r^2}$　　　　$\frac{G m_1 m_2}{r^2}$

単位電荷



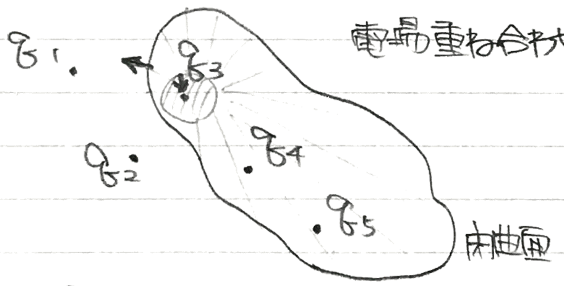
$$\frac{kq}{\|x\|^2}\frac{x}{\|x\|} = \frac{kqx}{\|x\|^3}$$

$$x = \begin{pmatrix}x\\y\\z\end{pmatrix} \longmapsto kq\begin{pmatrix}x\\y\\z\end{pmatrix}(x^2+y^2+z^2)^{-\frac{3}{2}}$$

$$\int_\sigma f\cdot dS = \underbrace{\int_\Omega (\mathrm{div}\, f)\, dV}_{=0}$$

$$\frac{kq}{a^2}\times 4\pi a^2 = 4\pi kq$$

電場の重ね合わせの原理



$$f = f_1 + f_2 + f_3 + f_4 + f_5$$

$$\int_\sigma f\, dS = \int_\sigma (f_1 + \cdots + f_5)\, dS$$

$$= \underbrace{\int_\sigma f_1\, dS + }_{=0} \int_\sigma f_5\, dS$$

連合単　$\sigma \cup \sigma_3$　$\int_{\sigma\cup\sigma_3} f_3 \cdot dS = 0$

$$\int_\sigma f_3\, dS - \int_{\sigma_3} f\, dS = 0$$

$$\int_\sigma f_3\, dS = \int_{\sigma_3} f\, dS = 4\pi k q_3$$

$$4\pi k q_3 + 4\pi k q_4 + 4\pi k q_5$$
$$= 4\pi k(q_3 + q_4 + q_5)$$

対称

$4\pi a^2 \times f = 4\pi k q$　(ガウスの法則)

$$f = \frac{kq}{a^2}$$

2次の交代形式　$a, b \in \mathbb{R}^3$

$$\mathbb{R}^3 \times \mathbb{R}^3 \xrightarrow{\varphi} \mathbb{R}^4$$

$$(\varphi\wedge\psi)(a, b)$$
$$= \varphi(a)\psi(b) - \varphi(b)\psi(a)$$

$$(\varphi\wedge\psi)(a_1 + a_2, b)$$
$$= \varphi(a_1 + a_2)\psi(b) - \varphi(b)\psi(a_1 + a_2)$$
$$= \{\varphi(a_1) + \varphi(a_2)\}\psi(b) - \varphi(b)\{\psi(a_1) + \psi(a_2)\}$$
$$= \{\varphi(a_1)\psi(b) - \varphi(b)\varphi(a_1)\} + \{\varphi(a_2)\psi(b) - \varphi(b)\psi(a_2)\}$$
$$= (\varphi\wedge\psi)(a_1, b) + (\varphi\wedge\psi)(a_2, b)$$

report I

$$(\varphi\wedge\psi)(\alpha a, b) = \alpha(\varphi\wedge\psi)(a, b)$$

report II

$$(\varphi\wedge\psi)(a, b) = -(\varphi\wedge\psi)(b, a)$$

$$(dx\wedge dy)(a, b) = dx(a)dy(b) - dx(b)dy(a)$$
$$= a_1 b_2 - b_1 a_2$$
$$= \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

$$b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$$a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$$