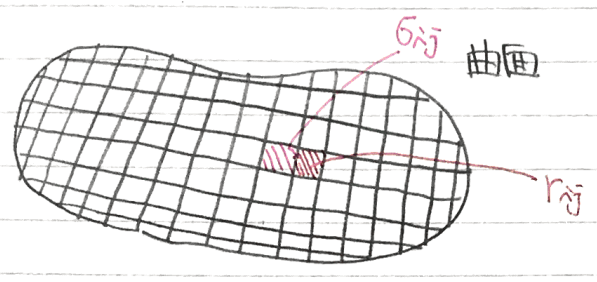


第18回 1/5



$$\int_{\sigma} (\text{rot } f) dS = \sum_i \sum_j \int_{\sigma_{ij}} (\text{rot } f) dS$$

$$= \sum_i \sum_j \int r_{ij} f dt$$

交代形式
R³ 線型空間

$e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$a = a_1 e_1 + a_2 e_2 + a_3 e_3$

$\mathbb{R}^3 \rightarrow \mathbb{R}$
$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \mapsto a_1 dx$
$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \mapsto a_2 dy$
$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \mapsto a_3 dz$

$\varphi: \mathbb{R}^3 \rightarrow \mathbb{R}$ の線型関数
 $\varphi(a+b) = \varphi(a) + \varphi(b)$
 $\varphi(\alpha a) = \alpha \varphi(a)$

$a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = a_1 e_1 + a_2 e_2 + a_3 e_3$

$\varphi(a) = \varphi(a_1 e_1 + a_2 e_2 + a_3 e_3)$
 $= a_1 \varphi(e_1) + a_2 \varphi(e_2) + a_3 \varphi(e_3)$
 $= \varphi(e_1) dx(a) + \varphi(e_2) dy(a) + \varphi(e_3) dz(a)$

2次の交代形式
 $\varphi: \mathbb{R}^3 \times \mathbb{R}^3 \rightarrow \mathbb{R}$
 2重線型

$\varphi(a_1 + a_2, b) = \varphi(a_1, b) + \varphi(a_2, b)$
 $\varphi(\alpha a, b) = \alpha \varphi(a, b)$
 $\varphi(a, b) = -\varphi(b, a)$ ← 交代性 (antisymmetry)

$(a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}, b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}) \rightarrow \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix} = a_2 b_3 - a_3 b_2$

$\rightarrow \begin{vmatrix} a_3 & b_3 \\ a_1 & b_1 \end{vmatrix} = a_3 b_1 - a_1 b_3$ $dz \wedge dx$ 基底

$\rightarrow \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1$ $dx \wedge dy$

$\varphi(a, b) = \varphi(a_1 e_1 + a_2 e_2 + a_3 e_3, b_1 e_1 + b_2 e_2 + b_3 e_3)$
 $= a_1 b_1 \varphi(e_1, e_1) + a_2 b_2 \varphi(e_2, e_2) + a_3 b_3 \varphi(e_3, e_3)$
 $+ a_2 b_3 \varphi(e_2, e_3) + a_3 b_2 \varphi(e_3, e_2)$
 $+ a_3 b_1 \varphi(e_3, e_1) + a_1 b_3 \varphi(e_1, e_3)$
 $+ a_1 b_2 \varphi(e_1, e_2) + a_2 b_1 \varphi(e_2, e_1)$
 $= (a_2 b_3 - a_3 b_2) \varphi(e_2, e_3) + (a_3 b_1 - a_1 b_3) \varphi(e_3, e_1) + (a_1 b_2 - a_2 b_1) \varphi(e_1, e_2)$
 $= (dz \wedge dx)(a, b) + (dx \wedge dy)(a, b)$

report
 $(dx \wedge dy \wedge dz)(a, b, c)$
 $= |a, b, c|$

$dy \wedge dz$
 ← 基底