$$\nabla\cdot\boldsymbol{F} = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \cdot \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix}$$

内積

$$= \frac{\partial}{\partial x} f_1 + \frac{\partial}{\partial y} f_2 + \frac{\partial}{\partial z} \cdot f_3$$

report I

(1) スカラー場

$\mathrm{rot}(\mathrm{grad}\,\varphi) = 0$

(2) ベクトル場

$\mathrm{div}(\mathrm{rot}\,\boldsymbol{F}) = 0$

report II

$\boldsymbol{F} = (x, y, z) \mapsto (x, y, z)$

$|\boldsymbol{F}|\, H^{k}$.

このベクトル場の回転を計算せよ

$(x, y, z) \longmapsto (x, y, z)(x^2 + y^2 + z^2)^{\frac{k}{2}}$

$F' = f$

$\int_a^b f(x)dx = F(b) - F(a)$

ガウスの発散定理

閉曲面のベクトル場 $\boldsymbol{F}$　閉曲面 σ

閉曲面の領域 Ω

$$\underbrace{\int_\sigma \boldsymbol{F}\, dS}_{\text{面積分}} = \underbrace{\int_\Omega (\mathrm{div}\,\boldsymbol{F})\, dV}_{\text{体積分}}$$

stokesの定理

σを閉曲線γを縁とする曲面.

$\boldsymbol{F}$: ベクトル場

$$\int_\gamma \boldsymbol{F}\cdot d\boldsymbol{r} = \int_\sigma (\mathrm{rot}\,\boldsymbol{F})\cdot dS$$

線積分　面積分

φ: スカラー場

γ: 曲線 $[a, b] \to \mathbb{R}^3$

$$\varphi(\gamma(b)) - \varphi(\gamma(a)) = \int_\gamma (\mathrm{grad}\,\varphi)\cdot d\boldsymbol{r}.$$

線積分

$$\varphi'(x) = \left[\frac{\partial\varphi}{\partial x}(x)\ \ \frac{\partial\varphi}{\partial y}(x)\ \ \frac{\partial\varphi}{\partial z}(x)\right]$$

線形写像

$f: \mathbb{R} \to \mathbb{R}$.

$f'(x) =$ $\mathbb{R}$から$\mathbb{R}$への線形写像　比例定数.

証明

$$\int_\gamma \mathrm{grad}\,\varphi \cdot d\boldsymbol{r}$$

$$= \int_a^b (\mathrm{grad}\,\varphi)(\gamma(t)) \cdot \gamma'(t)\, dt$$

内積

$$= \int_a^b \begin{pmatrix} \frac{\partial\varphi}{\partial x}(\gamma(t)) \\ \frac{\partial\varphi}{\partial y}(\gamma(t)) \\ \frac{\partial\varphi}{\partial z}(\gamma(t)) \end{pmatrix} \cdot \begin{pmatrix} \gamma_1'(t) \\ \gamma_2'(t) \\ \gamma_3'(t) \end{pmatrix} dt$$

$$= \int_a^b \left(\boxed{\frac{\partial\varphi}{\partial x}(\gamma(t))\cdot\gamma_1'(t) + \frac{\partial\varphi}{\partial y}(\gamma(t))\cdot\gamma_2'(t) + \frac{\partial\varphi}{\partial z}(\gamma(t))\cdot\gamma_3'(t)} \right) dt$$

合成関数を微分したもの

$\gamma: [a, b] \to \mathbb{R}^3$

$\varphi: \mathbb{R}^3 \to \mathbb{R}$

$(\varphi \circ \gamma)(t) = \varphi(\gamma(t))$

tで微分：合成関数の微分

$\int_a^b f(x)dx = F(b) - F(a)$

$\int_a^{a+d} f(x)dx \fallingdotseq f(a)d.$

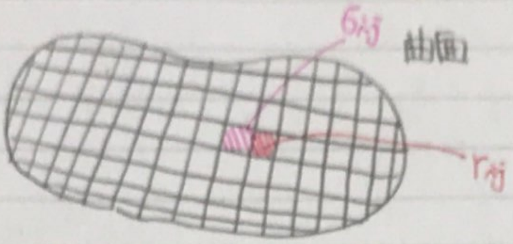
$= f(a+d) - F(a)$

$\int_{a_i}^{a_{i+1}} f(x)dx = F(a_{i+1}) - F(a_i)$

$F(a_{i+2}) - F(a_{i+1})$

$\vdots$

$= F(a_n) - F(a_i)$



$$\int_\sigma (\mathrm{rot}\,\mathbf{f})\,dS = \sum_i \sum_j \int_{\sigma_{ij}} (\mathrm{rot}\,\mathbf{f})\,dS$$
$$= \sum_i \sum_j \int_{r_{ij}} \mathbf{f}\,d\mathbf{r}$$

交代形式

$\mathbb{R}^3$ 線型空間

$\mathbf{e}_1 = \begin{pmatrix}1\\0\\0\end{pmatrix},\ \mathbf{e}_2 = \begin{pmatrix}0\\1\\0\end{pmatrix},\ \mathbf{e}_3 = \begin{pmatrix}0\\0\\1\end{pmatrix}$

$\mathbf{a} = a_1\mathbf{e}_1 + a_2\mathbf{e}_2 + a_3\mathbf{e}_3$

基底(base)

$\mathbb{R}^3 \to \mathbb{R}$

$\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix} \mapsto a_1 \quad dx$

$\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix} \mapsto a_2 \quad dy$

$\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix} \mapsto a_3 \quad dz$

$\varphi: \mathbb{R}^3 \to \mathbb{R}$ の線型関数

$\varphi(\mathbf{a}+\mathbf{b}) = \varphi(\mathbf{a}) + \varphi(\mathbf{b})$

$\varphi(\alpha\mathbf{a}) = \alpha\varphi(\mathbf{a})$

$\mathbf{a} = \begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix} = a_1\mathbf{e}_1 + a_2\mathbf{e}_2 + a_3\mathbf{e}_3$

$\varphi(\mathbf{a}) = \varphi(a_1\mathbf{e}_1 + a_2\mathbf{e}_2 + a_3\mathbf{e}_3)$

$= a_1\varphi(\mathbf{e}_1) + a_2\varphi(\mathbf{e}_2) + a_3\varphi(\mathbf{e}_3)$

$= \varphi(\mathbf{e}_1)\boxed{dx(\mathbf{a})} + \varphi(\mathbf{e}_2)\boxed{dy(\mathbf{a})} + \varphi(\mathbf{e}_3)\boxed{dz(\mathbf{a})}$

2次の交代形式

$\varphi: \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}$

2重線型

$\varphi(\mathbf{a}_1 + \mathbf{a}_2, \mathbf{b}) = \varphi(\mathbf{a}_1, \mathbf{b}) + \varphi(\mathbf{a}_2, \mathbf{b})$

$\varphi(\alpha\mathbf{a}, \mathbf{b}) = \alpha\varphi(\mathbf{a}, \mathbf{b})$

$\varphi(\mathbf{a}, \mathbf{b}) = -\varphi(\mathbf{b}, \mathbf{a})$ ← これが成り立つのは $\varphi(\mathbf{a}, \mathbf{b})$が0だから

$\left(\mathbf{a} = \begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}, \mathbf{b} = \begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}\right) \longrightarrow \begin{vmatrix}a_2 & b_2\\a_3 & b_3\end{vmatrix} = a_2b_3 - a_3b_2$

$dy \wedge dz$

くさび形

$\to \begin{vmatrix}a_3 & b_3\\a_1 & b_1\end{vmatrix} = a_3b_1 - a_1b_3 \quad dz \wedge dx$ 基底

$\to \begin{vmatrix}a_1 & b_1\\a_2 & b_2\end{vmatrix} = a_1b_2 - a_2b_1 \quad dx \wedge dy$

$\varphi(\mathbf{a}, \mathbf{b}) = \varphi(a_1\mathbf{e}_1 + a_2\mathbf{e}_2 + a_3\mathbf{e}_3,\ b_1\mathbf{e}_1 + b_2\mathbf{e}_2 + b_3\mathbf{e}_3)$

$= a_1b_1\underbrace{\varphi(\mathbf{e}_1, \mathbf{e}_1)}_{=0} + a_2b_2\underbrace{\varphi(\mathbf{e}_2, \mathbf{e}_2)}_{=0} + a_3b_3\underbrace{\varphi(\mathbf{e}_3, \mathbf{e}_3)}_{=0}$

$+ a_2b_3\varphi(\mathbf{e}_2, \mathbf{e}_3) + a_3b_2\underbrace{\boxed{\varphi(\mathbf{e}_3, \mathbf{e}_2)}}_{-\varphi(\mathbf{e}_2, \mathbf{e}_3)}$

$+ a_3b_1\varphi(\mathbf{e}_3, \mathbf{e}_1) + a_1b_3\underbrace{\boxed{\varphi(\mathbf{e}_1, \mathbf{e}_3)}}_{-\varphi(\mathbf{e}_3, \mathbf{e}_1)}$

$+ a_1b_2\varphi(\mathbf{e}_1, \mathbf{e}_2) + a_2b_1\underbrace{\boxed{\varphi(\mathbf{e}_2, \mathbf{e}_1)}}_{-\varphi(\mathbf{e}_1, \mathbf{e}_2)}$

$(dy \wedge dz)(\mathbf{a}, \mathbf{b})$

$= \boxed{(a_2b_3 - a_3b_2)}\varphi(\mathbf{e}_2, \mathbf{e}_3)$

$(dz \wedge dx)(\mathbf{a}, \mathbf{b})$

$+ \boxed{(a_3b_1 - a_1b_3)}\varphi(\mathbf{e}_3, \mathbf{e}_1)$

$+ \boxed{(a_1b_2 - a_2b_1)}\varphi(\mathbf{e}_1, \mathbf{e}_2)$

$(dx \wedge dy)(\mathbf{a}, \mathbf{b})$

report

$(dx \wedge dy \wedge dz)(\mathbf{a}, \mathbf{b}, \mathbf{c})$

$= |\mathbf{a}, \mathbf{b}, \mathbf{c}|$