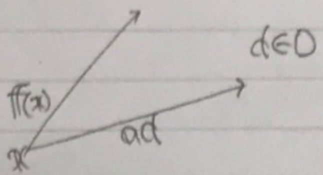
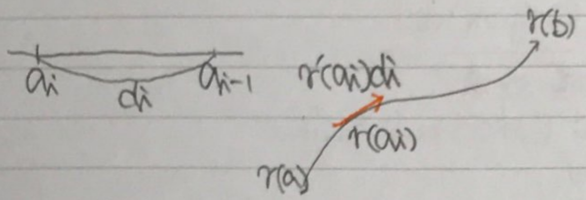
Date 10/18



$\boldsymbol{f}(x)\cdot ad=(\boldsymbol{f}(x)\cdot a)d$

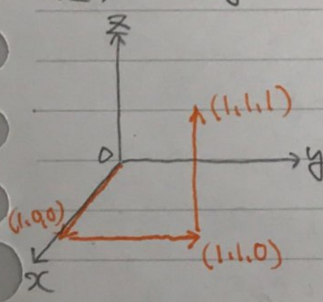


$$\sum_{i=0}^{n-1}\boldsymbol{f}(r(a_i))\cdot r'(a_i)d_i=\int_a^b \boldsymbol{f}(r(t))\cdot r'(t)\,dt$$

report I

ベクトル場

$$\begin{pmatrix}x\\y\\z\end{pmatrix}\mapsto\begin{pmatrix}x^2\\y\\xyz\end{pmatrix}$$



$r_1: t\in[0,1]\mapsto(t,0,0)$

$r_2: t\in[0,1]\mapsto(1,t,0)$

$r_3: t\in[0,1]\mapsto(1,1,t)$

$r: t\in[0,1]\mapsto(t,t,t)$

$r_1'(t)=(1,0,0)$

$\boldsymbol{f}(r_1(t))=\begin{pmatrix}t^2\\0\\0\end{pmatrix}$

$\boldsymbol{f}(r_1(t))\cdot r'(t)=\begin{pmatrix}t^2\\0\\0\end{pmatrix}\cdot\begin{pmatrix}1\\0\\0\end{pmatrix}=t^2$

$\int_0^1\boldsymbol{f}(r_1(t))\cdot r'(t)\,dt=\int_0^1 t^2\,dt=\frac{1}{3}$

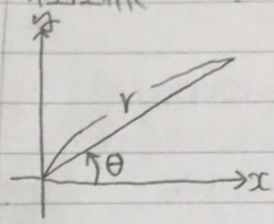
面積分

$\boldsymbol{f}:\mathbb{R}^3\to\mathbb{R}^3$：ベクトル場(流れの場)

曲面

$[a_1,a_2]\times[b_1,b_2]\longrightarrow\mathbb{R}^3$

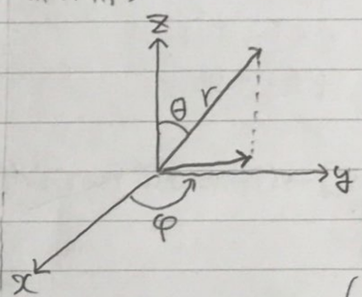
極座標



$x=r\cos\theta$

$y=r\sin\theta$

正射影



(r,θ,φ)

$r\in[0,\infty]$

$\theta\in[0,\pi]$

$\varphi\in[0,2\pi]$

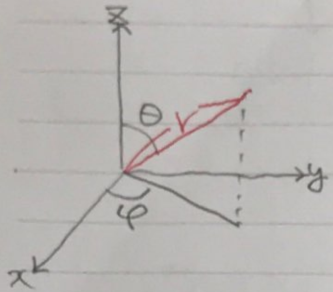
$$(\theta,\varphi)\in[0,\pi]\times[0,2\pi]\mapsto\begin{pmatrix}a\sin\theta\cos\varphi\\a\sin\theta\sin\varphi\\a\cos\theta\end{pmatrix}$$

$$\begin{cases}x=r\sin\theta\cos\varphi\\y=r\sin\theta\sin\varphi\\z=r\cos\theta\end{cases}$$

体積分

空間の領域 $\Omega:=[a_1,a_2]\times[b_1,b_2]\times[c_1,c_2] \to \mathbb{R}^3$

(例) 極座標 / 直交座標 } 正射影



$$\begin{cases} x=r\sin\theta\cos\varphi \\ y=r\sin\theta\sin\varphi \\ z=r\cos\theta \end{cases}$$

$r\geq 0$

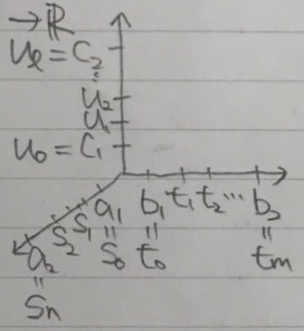
$\theta\in[0,\pi]$

$\varphi\in[0,2\pi]$

$$\underset{r}{[0,a]}\times\underset{\theta}{[0,\pi]}\times\underset{\varphi}{[0,2\pi]}\to(r\sin\theta\cos\varphi,\ r\sin\theta\sin\varphi,\ r\cos\theta)$$

スカラー場

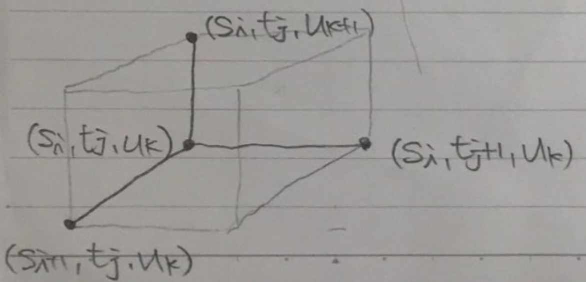
$\varphi:\mathbb{R}^3\to\mathbb{R}$



$s_{i+1}-s_i=d_i\in D$

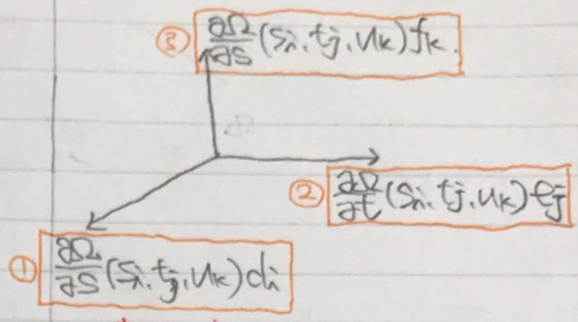
$t_{j+1}-t_j=e_j\in D$

$u_{k+1}-u_k=f_k\in D$



$\mathbb{R}^3$ 平行六面体

$\Omega(s_i,t_j,u_k)$



$$\sum_{k=0}^{l-1}\sum_{j=0}^{m-1}\sum_{i=0}^{n-1} ① \overset{\text{内積}}{\cdot} (② \overset{\text{外積}}{\times} ③)\, d_i e_j f_k$$

$$=\int_{c_1}^{c_2}\int_{b_1}^{b_2}\int_{a_1}^{a_2}\varphi(\Omega(s,t,u))\left\{\frac{\partial\Omega}{\partial s}(s,t,u)\cdot\left(\frac{\partial\Omega}{\partial t}(s,t,u)\times\frac{\partial\Omega}{\partial u}(s,t,u)\right)\right\}ds\,dt\,du$$

sに関して定積分
↓
tに関して定積分
↓
uに関して定積分

report問題

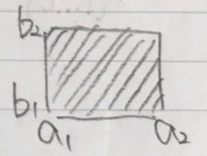
半径aの球の体積を求めよ.

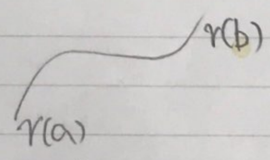
線積分

$\gamma[a,b]\to\mathbb{R}^3$

F: ベクトル場

$$\int_{b_1}^{b_2}\int_{a_1}^{a_2}f(x,y)\,dx\,dy$$





曲面、面積分、ベクトル場F(水の流れの場)

曲面σ

$\gamma[a,b]\to\mathbb{R}^3$

$\gamma(a)=\gamma(b)$

(閉曲線、閉曲面)